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1930-1937: The First β-ray and Neutrino Theories

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Abstract: The conceptual bases of Fermi's β-ray theory (at its 90th anniversary) are examined, highlighting the innovative drive and inspirational role for the progress that followed just afterwards. Moreover, the three different ideas of the neutrino born from the proposals of Pauli 1930, Fermi 1933 and Majorana 1937 are discussed, emphasizing the interest of the latter for current expectations.

Keywords: Nuclear physics, beta decay, neutrino, Pauli, Fermi, Majorana

1. Introduction

In 1984, on the 50th anniversary of the discovery of radioactivity induced by neutrons, Edoardo Amaldi wrote a monumental work of review on those very topics (Amaldi 1984). This work contains much valuable and unique material: e.g., there is a famous footnote, in which the origin of the word "neutrino" is recounted. We do not quote its text, relying on the fact that this story is already known, and we deal instead with Amaldi's presentation of an important and closely related aspect. In the section entitled "Fermi's paper on beta decay" (page 82) there is a description that leads the modern reader to spontaneous assent, this one:

> his density of interaction Hamiltonian H_fi is expressed as the product of 2 four-vectors computed at the same point (contact interaction), one concerning the heavy particles, the other the light particles
>
> $$H_{fi} = g[(\underline{\psi_p}\gamma_\mu\psi_n)(\underline{\psi_e}\gamma^\mu\psi_\nu) + h.c.]$$

On the other hand, scrolling the text of the three original work (Fermi 1933-1934), it is easy to convince oneself that Fermi uses a Hamiltonian, not a 'Hamiltonian density'; that his description of nucleons does not rely on the relativistic formalism (and in this way, the symmetry between hadrons and leptons is not emphasized); that neither Dirac γ_μ matrices nor Dirac conjugates are mentioned; that there is no mention of an emission of antineutrinos, but only of neutrinos. In short, the equation shown by Amaldi is not in Fermi's papers. It is a modern expression, to which modern theoretical physicists are accustomed, and which in a certain sense corresponds to those in (Fermi 1933-1934); but that does not allow us to understand the difficulties encountered and overcome by Fermi, and that also prevents us from appreciating the value of subsequent theoretical progress.

In view of the fact that Amaldi's review paper has been (and is) influential, and presentations similar or identical to his have since become very common – see e.g., (Bilenky 2013) – we propose to consider a series of questions to prepare ourselves to better appreciate Fermi's work and legacy:

- What are the objectives and conceptual bases of Fermi's theory? What are its radical innovations?
- What was Fermi's theory important for at the time?
- In what aspects does Fermi's theory of β decay differ from the modern one?
- How do Pauli's, Fermi's and Majorana's ideas on the neutrino compare with each other?

In the following discussion, we will draw mainly on a recent article prepared on the occasion of the 90th anniversary of Fermi's 'Tentativo' (Vissani 2023) to which we refer the reader, interested in detailed information and specific references.

2. Fermi theory of β rays and its legacy

2.1 Origin, purpose, basis and innovations

The aim of Fermi's work is to provide an answer to the question "how is it possible for the nucleus to emit electrons, if there are no electrons in the nucleus?" The formulation of this question helps us to remind the state of previous knowledge: in the second decade of the 20th century, a somewhat spontaneous opinion gained traction, that the electrons emitted in the β decay must pre-exist in the nucleus. This view is clearly stated in a well-known work by Rutherford written in 1920, in which he adheres to a model of a nucleus consisting of protons and electrons. As soon as the neutron is discovered, a new and more convincing model is proposed, where the nucleus contains only protons and neutrons (Iwanenko 1932, Heisenberg 1932, Majorana 1933); but this urgently raises the question of how to model the emission β rays.

Inspired by de Broglie's ideas, Ambarzumian & Iwanenko had suggested already in 1930 that the electron is created in that process, just as happens to a photon spontaneously emitted by an excited atom; the same was further advocated in 1933 by Francis Perrin, for whom the neutrino should also suffer a similar fate. But none of them succeeded in creating a quantitative theory, a calculable model.

Fermi, on the other hand, succeeded in this endeavour, with the three papers mentioned above, which have the same content. The first of these appeared just 90 years ago, and the other two provide some further details. The model describes the situation in which an atomic nucleus increases its charge by one unit (attributing this to a change of state of a nucleon – from a neutron, to a proton) and at the same time an electron and a neutrino are created. In formulae,

$$(A, Z) \rightarrow (A, Z + 1) + e + v$$

Let us immediately remember that Fermi's description is overall in good agreement with the observational facts; over time it has been improved in various aspects, rather than radically modified. But let's take a closer look at its original structure.

The mathematical formalism adopted to deal with relativistic fermions assumes the correctness of the Dirac equation, the Dirac sea-based interpretation, and exploits the technique of second quantization developed by Jordan, Klein, Wigner and Fock. This formalism implies using operators

$$\boldsymbol{\Psi} = \sum_{s} \boldsymbol{a}_s \psi_s$$

with dimensions square root of a density; the sum is over all possible states S (positive and negative energies); ψ_s are wavefunctions that solve Dirac equation, normalized *à la* Born; $\mathbf{a}_s$ are adimensional annihilation operators that describe the disappearance of a particle in the state s: $\langle 0|\mathbf{a}_s|s\rangle = 1$.

How to avoid a disastrous process of creating electrons of negative energy? The chosen way to go is the one described by Dirac. It is assumed that, as a rule, all negative energy fermion states are occupied; this is the hypothesis of the "Dirac sea". We reiterate that this formalism is used *only* for electrons and neutrinos. In this way, Fermi

- manages to describe the spin of electrons and neutrinos in the theory;
- does not emphasize the other crucial aspect of Dirac equation – antiparticles;

- relies on the less innovative – but adequate – isospin formalism for nucleons.

Let us emphasize the point we made, as explicitly as possible: Fermi uses quantized fields to deal with electrons and neutrinos, but not the formalism of canonical quantization.
The original form of Fermi's hamiltonian is the following one,

$$\mathcal{H} = g\boldsymbol{Q}(\boldsymbol{\Psi}^t \delta \boldsymbol{\Phi}) + h.c.$$

- g denotes Fermi's constant, with units energy per volume;
- **Q** the dimensionless isospin matrix, which transforms a proton into a neutron;
- $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ the fields of second quantization of the relativistic particles with spin 1/2, the electron and the neutrino, which have the same units as the wave functions (root of a density=root of an inverse volume);
- the superscript t denotes the transpose;
- δ a 4 × 4 dimensionless matrix which ensures the Lorentz invariance of the expression.

This is inspired by the interaction energy of electromagnetism $H = eV$: the scalar field V is replaced by the lepton current $\boldsymbol{\Psi}^t\delta\,\boldsymbol{\Phi}$. Note that the expression is given in the limit of nucleons at rest, and the hermitian conjugate is needed for probability conservation. (See Vissani (2023) for more discussion). From a conceptual point of view, the main innovation of Fermi's model is that it formally describes the possibility that a particle can be destroyed or created. It is the first time that *particles of matter* are assumed to undergo a similar fate. This constitutes a milestone for the beginning of modern particle physics, although the formalism adopted (which derives from Jordan, Klein on the one hand and from Dirac's positron theory on the other) does not coincide with the current one.

2.1 Reactions to Fermi's paper and its legacy

Fermi's use of Dirac sea exposed him at the time to the same criticism as Dirac.[1] In addition to these reservations of a general nature, the work will be the subject of a lively debate. Limiting ourselves for the moment to the main contributions of a critical nature, let us mention for example the specific 1935 proposal by Konopinski & Uhlenbeck, which at first seemed superior to Fermi's, but which emerges defeated from the confrontation a few years later. Then recall a criticism by Pauli in 1938, centered on the fact that the theory includes the parameter g with canonical dimensions equal to the inverse of a square mass in natural units, a circumstance that entails going outside the theory itself with perturbative orders higher than the first; but as is well known today, Fermi's theory is to be thought of as an effective theory and therefore is to be used precisely at the first perturbative order.

In short, Fermi's theory fully hits the mark, in spite of the usage of second quantization based on Dirac sea (or in Fermi's words, the Dirac, Jordan, Klein procedure) and the specific choice of Hamiltonian function, aspects that only apparently are limiting. To convince oneself of this, one need only recall three important works inspired by the 'Tentativo' and written soon afterwards, in 1934, by Wick (4 March); Bethe and Peierls (7 April); Yukawa (17 November):

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[1] See e.g., Pais (1986) and Kragh (1990). In his memoirs, Occhialini reiterates that the old guard physicists such as Rutherford and Bohr, but also Chadwick, maintained reservations at least until 1932. From Majorana's correspondence, a feeling of doubt towards Dirac interpretation persisted until 1933. L. Brown mentions Landau and Fock among the sceptics, and recalls Pauli's reservations towards Dirac argument to predict the positron continue in 1933; next year, Weisskopf and Pauli will succeed in quantizing a hypothetical spinless particle without resorting to Dirac sea – a procedure Pauli liked to refer to as 'anti-Dirac theory'. For references, see: Vissani (2023).

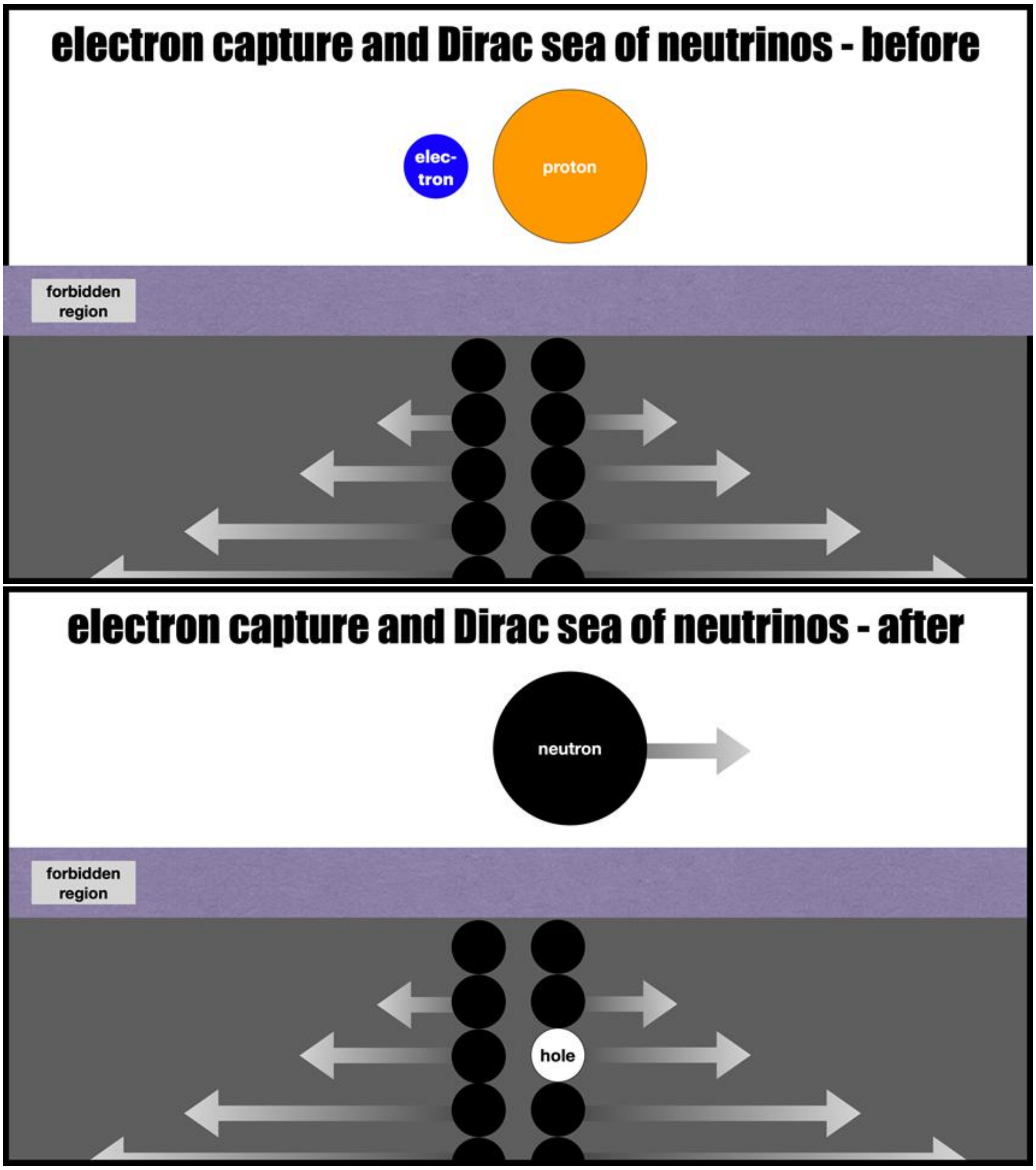

Fig. 1. Description of electron-proton capture in the formalism of second quantization (Wick 1934), emphasizing the Dirac sea of neutrinos (= region of negative energies). **Panel above**: Initial state of the process; an electron and a proton at rest can be seen; the neutrinos states of Dirac's sea are all occupied. **Panel below**: Final state of the process. The nucleon changed its isospin state and became a neutron; a hole has formed in the Dirac sea, which can be thought of as an antineutrino, moving in the opposite direction of the neutron (= Dirac hole theory).

1. *Wick* derives the predictions for β^+ emission and electron capture, using – just like Fermi – the second quantization formalism. The first process explains observations already obtained by Joliot and Curie, the second (one of the proofs for the existence of the neutrino) will receive experimental confirmation a few years later. See Fig.1 for an illustration of the latter process.
2. *Bethe & Peierls*, making explicit reference to (Fermi 1933), estimate the neutrino-nucleon interaction cross section by means of a brilliant argument. This reaction will be exploited for the first experimental observation of the neutrino.

3. *Yukawa*, interested in understanding interactions between nucleons, will propose the idea that interactions between nucleons and those between leptons are mediated by a boson with non-zero mass, in order to reproduce Fermi's theory by mimicking the structure of electromagnetic interactions.

2.3 Subsequent progresses of β decay theory

Nowadays, most particle physicists are aware of certain results of the theory of weak interactions, due to subsequent theoretical developments. For example, it is generally recalled that Gamow & Teller's in 1936 included the effect of spin in the nucleonic current, which using the current language of © matrices (Pauli 1936) we attribute to the presence of axial currents; even better known is the much later history of how the *V-A* structure (chiral interactions) of the charged currents was understood – see e.g., the fine work of review by Weinberg, written in 2009. Among the other recent developments we mention at least the understanding of the conservation of leptonic number in the *β* interactions, and the thorough examination of the structure of currents concluded and completed with the Cabibbo theory. All these advances dovetail and harmonise with Fermi's theory.

Here we would like to limit ourselves to highlighting an advance that occurred in the 1930s, for the simple reason that it is not sufficiently appreciated today. We refer to the procedure of quantization of fermionic fields due to another of the boys from via Panisperna, Ettore Majorana. In the first part of the summary of his work of 1937 (the last one) we read

> It is shown how to achieve a full formal symmetrization of the quantum theory of the electron and positron by making use of a new quantization process. The meaning of the equations of DIRAC equations is quite modified and there is no longer any need to speak of states of negative energy (Majorana 1937, p. 171).

Apart from a witty choice of basis for γ matrices used, the new procedure of quantization of fermions in Majorana 1937 is exactly the one used today, i.e., the 'canonical quantization'. To ascertain Fermi's appreciation of this result, read his judgement for the chair competition, held in the same year:

> [Majorana] devised a brilliant method for treating the positive and negative electron symmetrically, finally eliminating the need to resort to the extremely artificial and unsatisfactory hypothesis of an infinitely large electric charge spread throughout space, an issue that had been addressed in vain by many other scholars (Majorana 1937, p. 171).

If the terms were used literally, only from this moment on would it be legitimate to speak of a "vacuum state" rather than a "fundamental state". In more, evocative terms we can say that it was Majorana who showed the world *how to empty the Dirac sea.*[2] But an unaware reader, who believed that the notations in (Amaldi 1984) are the original ones would not even notice this step forward; and (losing sight of the context) he/she would no longer be able to truly understand Fermi's work.

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[2] It should be stressed that the Dirac sea hypothesis, unattractive from a physical point of view and now abandoned, is accompanied by relatively simple and almost spontaneous expressions for the second quantization fields. For this reason it maintains a certain interest in learning paths: it allows us to appreciate how we arrived at modern quantized field theory.

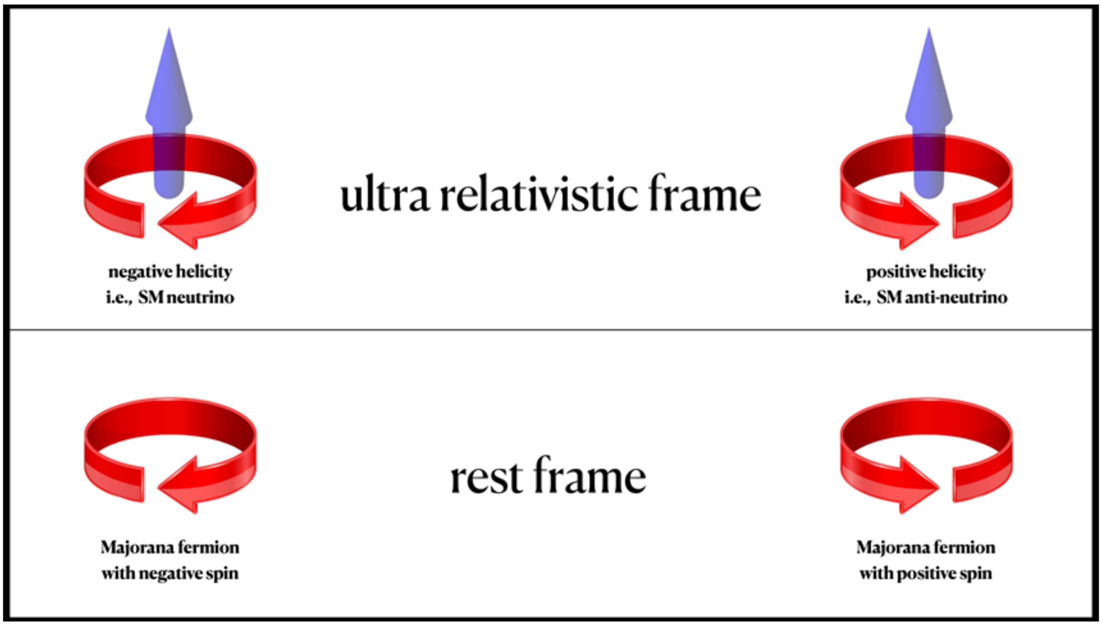

Fig. 2. Illustration of the concept of (neutrino with) Majorana mass in the context of the electroweak/V-A theory/standard model. The projection of spin onto the momentum of the particle – helicity – makes it possible to univocally tell neutrinos from antineutrinos in the ultra-relativistic limit. But in the rest system – which for massive fermions exists – the two states are identical, up to the orientation of the spin.

3. Pauli, Fermi and Majorana: three ideas on the neutrino compared

In this last section, we address one last conceptual point, and discuss the three different ideas of the neutrino that were formulated in the 1930s:

1. *Pauli 1930* introduced the neutrino as a constituent of the atomic nucleus in 1930 and assumed that this particle is emitted in β decay. This model has no relativistic characteristics and in particular has no connection with Dirac idea of antimatter.
2. *Fermi 1933-1934*, on the other hand, describes neutrinos that are relativistic fermions, completely analogous to the electron. Given the formalism adopted (which requires a Dirac sea of neutrinos with negative energy) antineutrinos exist and are quite distinct from neutrinos: see again Fig.1. (In other words, such a neutrino concept corresponds closely to what is now called the ‘Dirac neutrino’. Although this term is widespread today, Fermi does not use it and there is no work by Dirac describing such a neutrino concept.)
3. Finally, *Majorana 1937* neutrino idea is still different, and consists of the assumption that the neutrino and the antineutrino are the same particle. A similar identification applies for example to the photon, which however, unlike the neutrino, is not a particle of matter.

Here is how Majorana concludes the summary of his work:

> there is no longer reason [...] to assume for any other type of particles, particularly neutral ones, the existence of antiparticles corresponding to vacua of negative energy (Esposito *et al.* 2009, p. XIII).

where we note the statement on neutral particles which makes implicit reference to neutrinos.

We conclude by remarking that the structure of the “standard model” of the electroweak interactions – and in particular, the chiral nature of the charged-currents weak interactions and the way in which neutrinos are included – suggests that Majorana’s hypothesis is realized in nature, albeit in a quite specific way: the neutrino and the antineutrino, which we know to be different from each other when they move in ultra-relativistic motion, manifest themselves as the same particle in the system at rest. Fig. 2 better illustrates the physical content of this statement. This hypothesis on the nature of the neutrino is the subject of lively experimental investigations in laboratories all over the world.[3]

For a more detailed discussion and further references, we refer the reader gain to (Vissani 2023); for the modern developments of Fermi’s theory, see (Barbieri 2023).

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[3] For two recent reviews on this subject, see Vissani (2021) and Agostini *et al.* (2023). For an interesting discussion of the influence of Hermann Weyl’s ideas in neutrino physics, see De Bianchi (2018).

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