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Exchange Interactions between Europe and Japan in the 1930s: Tomonaga, Yukawa and the Birth of Nuclear Theory

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Abstract: The concept of exchange interactions was introduced by W. Heisenberg in 1926 in connection with the quantum mechanical description of systems of identical particles, and it was soon fruitfully applied to many problems in atomic, molecular, and condensed matter physics. After the discovery of the neutron in 1932, it found application also in nuclear physics, with the theories of nuclear structure developed, among others, by Heisenberg and E. Majorana. Then, it entered the domain of quantum field theory, leading to the modern understanding of fundamental forces as mediated by virtual particle exchange. An important intermediate step in this development is Fermi's theory of β-decay. H. Yukawa and S. Tomonaga, who already had been exposed to the principles of the new quantum mechanics by attending a series of lectures given by Heisenberg and P. A. M. Dirac in Japan in 1929, and spent long periods in Europe, were strongly influenced by these works. Within a few years, Yukawa conceived his crucial idea of an interaction mediated by virtual mesons, while Tomonaga investigated the range of proton-neutron interactions. In this contribution, we reconstruct the role played by Japanese physicists in building the modern understanding of fundamental forces in the 1930s and relate it to research performed in Europe.

Keywords: Nuclear Physics, Exchange Forces, Meson

1. Introduction

The birth of the idea of quantum resonance as the first seed of the so-called exchange interactions can be traced back to a 1926 seminal paper by W. Heisenberg (1926). Here he discovered the existence of a close connection between the symmetry property of the wave function and the statistics of a particle in a many-body system and succeeded in explaining the splitting between singlet and triplet states in the spectrum of the helium atom. Exchange interactions soon entered the domain of molecular physics thanks to W. Heitler and F. London, who in 1927 laid down the foundations of the quantum theory of the homopolar chemical bond (Heitler & London 1927). Here, an intuitive notion of electrons literally exchanging places around the different nuclei was introduced, which is denoted with the German word *Austausch*. The meaning of the exchange concept thus gradually evolved from Heisenberg's original idea, simply involving a switch of two states with different energy levels. In molecules, the net result of the exchange of electrons is a force between the nuclei. The idea was soon applied to explain the strong spin-spin interaction at the basis of ferromagnetism (Heisenberg 1928), where exchange interactions are responsible for the overall alignment of individual spins (Carson 1996a). Another application was to electron-atom collisions, where the exchange of the incoming electron with one of the atomic electrons produces interference terms modifying the cross-section (Oppenheimer 1928).

As well known, the year 1932 marked the beginning of a new era in nuclear physics triggered by the discovery of neutron (Chadwick 1932), which deeply contributed to a correct understanding of the nuclear structure. Indeed, the hypothesis that the nucleus is built of protons and electrons lead to many inconsistencies, as for instance the wrong statistics of the ^{14}N nucleus and the violation of the momentum-position uncertainty relation for the confined electrons. Another puzzling issue concerned the continuous nature of the β-ray spectrum. These problems led N. Bohr to believe that quantum mechanics did not work at nuclear scales, as evocatively recalled by S. Tomonaga: “according to Bohr’s idea, the interior of a nucleus was a sanctuary that could not be penetrated by quantum mechanics” (Tomonaga 1997, p. 160). After J. Chadwick’s discovery, Heisenberg was able to bring quantum mechanics into the nuclear domain and provided a first description of the interaction between protons and neutrons in terms of an exchange mechanism (Heisenberg 1932a; Heisenberg 1932b; Heisenberg 1933), soon followed by E. Majorana’s improvements (Bassani 2006; Majorana 1933) and E. Fermi’s theory of β-decay (Fermi 1934; Wilson 1968). Thus, the concept of exchange force entered nuclear physics together with a fruitful analogy between quantum electrodynamics and nuclear forces, paving the way to the modern understanding of fundamental interactions as mediated by virtual particles. But the last, crucial step towards this picture was carried out in Japan by H. Yukawa. He, as well as his classmate and rival scientist Tomonaga, began to study the new quantum mechanics while being third year undergraduate students at Kyōto University (Yukawa 1982; Tomonaga 1997). Then in 1929, as unpaid research assistants, they attended a series of lectures given in Tōkyō and Kyōto by Heisenberg and Dirac and promoted by Y. Nishina, who had recently returned to Japan after an eight-year research stay in Europe (Kim 2007; Konagaya 2020). The meson idea took shape at the end of 1934 (Yukawa 1935) after a three-year struggle (Yukawa 1982) and a fruitful interaction with Tomonaga, who (in the same years) was carrying out calculations on the binding energy of the deuteron and the range of proton-neutron interactions in Nishina’s laboratory (Nishina *et al*. 1936; Tomonaga 1997).

The role played by Yukawa as well as Tomonaga in the birth of the theory of nuclear forces is here reconstructed and compared with that of European physicists. It is discussed in detail in Section 3 and 4 of the present paper, while Section 2 contains a brief account of the key ideas underlying the contributions by Heisenberg, Majorana and Fermi. Finally, our conclusions end the paper.

2. The birth of theoretical nuclear physics: Heisenberg, Majorana and Fermi

Heisenberg’s seminal papers (Heisenberg 1932a; 1932b; 1933) were the first concrete attempt to bring quantum mechanics into the nuclear domain. The starting point was the hypothesis that the neutron is a nuclear constituent, which was expected to lead to the correct statistics of the nitrogen nucleus as well as to an explanation of the β-decay mechanism. In his words:

> The neutron will be taken as an independent fundamental particle which, however, can split, under favourable conditions, into a proton and an electron, violating the law of conservation of energy and momentum (Heisenberg 1932, p. 1; transl. in Brink 1965, p. 145).

This hybrid vision, pointing sometimes to the neutron as a bound system, will accompany Heisenberg throughout all the three papers. The core of his proposal is a quantum mechanical description of the interaction between protons and neutrons, whose nature he aimed at establishing. As inferred by experiments, the required interaction should be very strong within the nuclear region and should satisfy a saturation property, in much the same way as the interaction between the two protons in a hydrogen molecular ion:

> If one brings a neutron and a proton to within a distance comparable to the dimensions of the nucleus, then – in analogy with the H_2^+ ion – a change of place of the negative charge will occur with a frequency given by a function *(1/h) J(r)* of the distance between the two particles. The quantity *J(r)* corresponds to the Austausch- or more correctly the Platzwechsel-integral of molecular

> theory. One can illustrate this change of place again with the picture of electrons that have no spin and obey Bose statistics. But it is probably more correct to regard the exchange integral *J(r)* as a fundamental property of the proton-neutron pair, without wanting to reduce it to motion of electrons (Heisenberg 1932, p. 2; transl. in Carson 1996b, p. 104).

Heisenberg referred to an electron without spin satisfying Bose statistics, hinting at a composite neutron. Thus, the interaction between the neutron and the proton appeared to be due to an exchange of an electron between them, as in the case of the H_2^+ molecular ion, a mechanism termed in German *Platzwechsel*, i.e., exchange of position (Carson 1996b; Miller 1994). But at the end a novel picture is suggested: if one forgets about the electron, the effect can be viewed as due to proton-neutron exchange, again dealing with an *Austausch* (Carson 1996b; Miller 1994). The last consideration led Heisenberg to introduce the successful concept of isospin, which allowed him to interpret the neutron and the proton as the two different states of a nucleon (Heisenberg 1932a). Without any doubt Heisenberg's theory contains deep insights, which allowed to penetrate the nuclear "sanctuary", but there were also several shortcomings. In particular, the ground state wave function of the deuteron was symmetric with respect to the isospin and space coordinates and antisymmetric with respect to spin as a result of the choice of the sign of *J*(*r*). This conclusion was wrong, leading to the saturation of the deuteron, at odds with experimental results, which pointed to the α-particle instead. As a further drawback, the extension of Thomas-Fermi method to nuclei (Heisenberg 1933) needed a cut-off to potential energy at short distances in order to avoid the collapse of the nucleus.

A critical analysis of Heisenberg's results was Majorana's starting point (Majorana 1933; Bassani 2006). The Italian physicist adopted a different approach in order to circumvent the shortcomings of Heisenberg's theory. He looked for the simplest theory able in principle to account for the neutron-proton interaction, which could explain basic facts such as the saturation of the α-particle and the correct spin of the deuteron ground state. Heisenberg's strategy of selecting out an attractive force for large distances cut-off by a strong repulsive short-range force to model impenetrability of particles was discarded as aesthetically unsatisfactory, thus Majorana pursued a different strategy, again inspired by molecular physics:

> We shall, therefore, try to find another solution and introduce as few arbitrary elements as possible. The main problem is this: How can we obtain a density independent of the nuclear mass without obstructing the free movement of the particles by an artificial impenetrability? We must try to find an interaction whose average energy per particle never exceeds a certain limit however great the density. This might occur through a sort of saturation phenomenon more or less analogous to valence saturation (Bassani 2006, p. 188).

Thus, Majorana corrected Heisenberg's theory by changing the sign of the exchange integral *J(r)* and by assuming that the exchange interaction exchanged only the position coordinates of proton and neutron while the spins were unaffected. This choice led to the saturation of the α-particle, as explicitly proven by applying the Thomas-Fermi model to nuclear matter. Finally, he looked for the best analytical form of *J*(*r*), which could match experimental data. Interestingly, his second proposal, $J(r) = Ae^{-\beta r}$, is very close to Yukawa's one (Yukawa 1935). The superiority of Majorana's approach was publicly acknowledged by Heisenberg (1934) during his speech at the Seventh Solvay Conference, held in October 1933, and gathered wide consensus among physicists. But, despite these brilliant findings, the problem of energy non conservation in β-decay remained still unsolved. A brilliant solution was provided by Fermi in 1934. His work builds on the analogy with the emission of a photon by a decaying atom and makes a clever use of Pauli's neutrino hypothesis, Heisenberg's isotopic spin framework and second quantization (Fermi 1934; Wilson 1968). Furthermore, Fermi assumed that the nuclear constituents are only neutrons and protons. In his words:

> in order to understand that emission is possible, we want to try to construct a theory of the emission of lightweight particles from the nucleus in analogy with the theory of emission of light quanta from an excited atom by the usual radiation process. In radiation theory, the total number of light quanta is not constant. Light quanta are created when they are emitted from an atom, and are annihilated when they are absorbed (Wilson 1968, p. 1151).

The analogy with the electromagnetic case clearly led to the possibility both for electrons and neutrinos to be created and annihilated. Fermi carried out perturbative calculations and compared his results with available experimental data, obtaining a good agreement. Immediately after the success of Fermi's theory, various people (Tamm 1934; Iwanenko 1934) tried to unify β-radioactivity and the nuclear force by looking at the latter as due to an exchange of an electron-neutrino pair (instead of a Bose electron), but the results did not match the experimental order of magnitude of neutron-proton coupling. The problem would have been solved by Yukawa at the end of 1934.

3. Yukawa and the meson

In order to reconstruct the genesis of Yukawa's work, it is worth making reference to his first publication, which dates back to 1932, and amounted to a Japanese translation of Heisenberg's papers, supplemented with a short introduction.[1] As clearly emerges from his words, a critical analysis of Heisenberg's work was the starting point of his path towards the formulation of meson theory:

> Though Heisenberg does not present a definite view on whether neutrons should be seen as separate entities or as combinations of a proton and an electron, this problem, like the β decay problem stated above, cannot be resolved with today's theory. And unless these problems are resolved, one cannot say whether the view that electrons have no independent existence in the nucleus is correct (Brown 1981, p. 122).

Quantum field theory also played a crucial role in the genesis of the meson idea, leading from a field of force to the corresponding particle, as recollected in his autobiography (Yukawa 1982). The intermediate steps in his investigation are documented by a couple of talks given in 1933 and 1934 at the annual meeting of the Physico-Mathematical Society of Japan, which did not result into any publication. However, some related documents (abstracts and manuscripts), which give valuable information on Yukawa's reasoning, are collected in the Yukawa Hall Archival Library (YHAL) at YITP, Kyōto University.[2] In particular, in the 1933 talk there is a first attempt to derive the proton-neutron interaction from the relativistic quantum mechanics of the exchanged electron. In this respect, it is interesting to quote a passage from an unpublished manuscript (Yukawa 1933a):

> Now we will adopt the viewpoint that the neutron is quantum mechanically an elementary particle. […] With this assumption alone, however, we cannot understand how the neutron is bound in nuclei. Some attraction must act between protons and neutrons, and between neutrons themselves. What is the reason for such attraction? The force between charged particles is, as well known, mediated by radiation; a charged particle emits radiation in its motion and this radiation affects the other charged particles; these particles absorb the radiation and change their state of motion. That is, charged particles create the electromagnetic field and are affected by it. In the same sense, we intend to consider the following: a neutron can emit an electron and change to a proton, and a proton can absorb an electron and change to a neutron. This fact is itself the cause of the interaction between

[1] The English translation of this introduction can be found in: Brown (1981, pp. 121-122).

[2] See Kawabe (1991) for an English translation of the main documents related to Yukawa's 1933 and 1934 talks.

proton and neutron; in other words, neutron and proton create the electron field, i.e., the field of the electron wave, and are affected by it (Kawabe 1991, pp. 251-252).

In this way Yukawa also expected to explain β-decay. But how to guess the behavior of the exchange integral $J(r)$? In the same manuscript (Yukawa 1933a) he admitted that:

> when we regard the neutron as an elementary particle, there immediately arises the question: What kind of force acts between the neutron and the charged particles? To solve this problem the most effective method at present might be to calculate the scattering of neutrons on nuclei by assuming a suitable interaction between neutrons and nuclei and comparing the results with experiment. In effect, one can assume a force decreasing rapidly with the distance (Massey, Proc. R. Soc. London A 138 (1932), 460) […] in any case we cannot justify the assumptions (Kawabe 1991, pp. 250-251).

There is a clear hint to a force rapidly decreasing with distance and a reference to a paper by H.S.W. Massey (1932b), which dealt with collisions of neutrons with matter. Here the neutron was considered as an atom of very high effective Z (to account for its small size), so that the following potential is introduced:

$$V(r) = e^2\left(\frac{1}{r} + \frac{Z}{a_0}\right)e^{-\frac{2Zr}{a_0}} \qquad (1)$$

where a_0 is the Bohr radius. As pointed out by Kawabe (1991), Massey's paper refers to a previous paper of him (Massey 1932a), in which a nuclear field of the form $V(r) = -\frac{Ae^{-\mu r}}{r}$ is considered, when calculating the anomalous scattering of α-particles. Both expressions, in particular the second one, show a behavior very similar to Yukawa's interaction, but Yukawa did not refer to the second paper by Massey (1932a). Thus, one is not able to say if he was aware of it. In conclusion, already in 1933 Yukawa was very close to the correct form of $J(r)$, but he did not understand its meaning and its consequences yet. Furthermore, his calculations gave in fact rise to a Coulomb-like form for the interaction energy, in sharp disagreement with the expected behavior (Yukawa 1933b):

> In any case, the practical calculation does not yield the looked-for result that the interaction term decreases rapidly as the distance becomes larger than $h/2\pi mc$ unlike what I wrote in the abstract of this talk (Kawabe 1991, p. 249).

Yukawa initially attributed the failure of his approach to the incompleteness of relativistic quantum mechanics, which demanded a reformulation. So, he turned to foundational issues and began to develop a kind of nonlocal field theory, which would later be an inspiring source for Tomonaga's super-many-time theory (Tomonaga 1946). This topic was also the subject of his 1934 talk at the annual meeting of the Physico-Mathematical Society of Japan. But there were no novelties in Yukawa's research until late spring 1934, when he learned about Fermi's work on β-decay and turned again to nuclear theory. After trying himself to explain nuclear forces by means of the exchange of a neutrino-electron pair, he changed his approach and proposed a brand-new field as the basis of the interaction:

> Now, such an interaction […] can be described by means of a field of force, just as the interaction between the charged particles is described by the electromagnetic field. […] In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon (Yukawa 1935, pp. 48-49).

Yukawa thus seeked a new field theory for the nuclear force and tried to infer its properties from the known properties of the latter. So, he assumed that the potential had the simple form $\pm g^2 \frac{e^{-\lambda r}}{r}$ and viewed it as the static, spherically symmetric solution of the equation:

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = 0 \tag{2}$$

Here g and λ are constants with the dimensions of an electric charge and of an inverse length, so that the range of forces is equal to $1/\lambda$. Then, by assuming an appropriate source, he solved the corresponding equation and completed his program of writing down the Hamiltonian for the neutron-proton system. The resulting Hamiltonian turns out to be equivalent to Heisenberg's one, with the above potential (with the minus sign) playing the role of the exchange integral $J(r)$. Then, Yukawa noticed that quantum field theory required this field to be associated to massive quanta, later dubbed mesons. Furthermore, he recognized that the range of the force is related to the mass of the quanta. Yukawa also assumed that his quanta coupled to light particles (electrons and neutrinos), in order to unify nuclear and Fermi interactions. His U-field should have been the analog of the electromagnetic scalar potential. The analog of the vector potential was not worked out in the 1935 paper because at the time no suitable relativistic equation was known. The generalization to a vector theory would have been carried out later, in 1937 (Yukawa *et al*. 1938), after the discovery of a particle compatible with meson in cosmic ray showers (Anderson & Neddermeyer 1936). This event marked the beginning of the interest of western physicists in Yukawa's results.

4. Tomonaga's contributions

In his 1935 paper, Yukawa explicitly acknowledged Tomonaga's contributions:

> using the Hamiltonian (10) for heavy particles, we can calculate the mass defect of H^2 and the probability of scattering of a neutron by a proton […]. These calculations were made previously, according to the theory of Heisenberg, by Mr. Tomonaga, to whom the writer owes much. A little modification is necessary in our case (Yukawa 1935, p. 52).

Tomonaga's role can be reconstructed from archival documents[3] as well as from Tomonaga's recollections (Tomonaga 1997). In April 1933, at the same annual meeting of the Physico-Mathematical Society of Japan attended by Yukawa, Nishina and Tomonaga presented the results of a calculation on the neutron-proton interaction. A potential identical with Yukawa's one was introduced and an estimate of the range of the interaction was also provided (Tomonaga 1997). The abstract of their talk was:

> We have analysed scattering of neutron by proton using Heisenebrg's theory on nuclear structure, assuming the shape of interactions between neutron and proton, and taking into account the mass defect of hydrogen 2. Our result has been compared with experimental results (Konuma *et al.* 2014, p. 2).

At Yukawa's request, he sent him a seven-page letter (Tomonaga 1933), in which he gave details of his calculations, involving several short-range phenomenological potentials. Also, here he mentioned the potential $A\frac{e^{-\lambda r}}{r}$. In fact, Tomonaga's computations of neutron-proton scattering led him to predict the existence of a new quantum state of the deuteron, as vividly expressed in his recollections:

[3] See the Yukawa Hall Archival Library (YHAL) at YITP, Kyoto University. Some documents are also available in the Archive of historical materials, Osaka University Yukawa Memorial and accessible via web.

> I found that such a level did not result from Heisenberg's exchange force alone, but if we added the exchange force proposed by Majorana […] such a level was possible. Therefore, I wanted to determine the ratio of the Heisenberg force and the Majorana force from this scattering experiment. […]. I succeeded […]. I was quite elated with this achievement and Professor Nishina was also satisfied, and our results were reported […] in Sendai, 1933 […]. However, […] Professor Nishina put off publishing this paper (Tomonaga 1997, p. 228).

The corresponding paper (Nishina *et al*. 1936) would have been published only later, in 1936.

5. Conclusions

The 1930s were a decade of huge development for theoretical nuclear physics. Fermi's and Yukawa's theories were the first quantum field theories other than quantum electrodynamics. The decisive steps towards a quantum field theory of the nuclear interaction were made by the new generation of Japanese physicists. Yukawa's assumptions were the validity of the principles of quantum theory inside the nucleus and the idea that nuclear interaction is a fundamental force just as electromagnetism. A fruitful interplay between Yukawa and Tomonaga in 1933-34 was pivotal in the proposal of meson theory. In particular, Tomonaga's results with Nishina played a role in the birth of Yukawa's theory, worth of further investigations.

Bibliography

Anderson, C.D. & Neddermeyer, S.H. (1936). "Cloud Chamber Observations of Cosmic Rays at 4300 Meters Elevation and Near Sea-Level", *Physical Review*, 50, pp. 993-994.

Bassani, G.F. & the Council of the Italian Physical Society (eds.) (2006). *Ettore Majorana Scientific Papers. On occasion of the centenary of his birth*. Berlin and Heidelberg: Springer Verlag.

Brink, D.M. (1965). *Nuclear Forces*. Oxford: Pergamon Press.

Brown, L.M. (1981). "Yukawa's Prediction of the Meson", *Centaurus*, 25, pp. 71-132.

Carson, C. (1996a). "The Peculiar Notion of Exchange Forces-I", *Studies in History and Philosophy of Modern Physics*, 27, pp. 23-45.

Carson, C. (1996b). "The Peculiar Notion of Exchange Forces-II", *Studies in History and Philosophy of Modern Physics*, 27, pp. 99-131

Chadwick, J. (1932). "The existence of a neutron", *Proceedings of the Royal Society of London A*, 136, pp. 692-708.

Fermi, E. (1934). "Tentativo di una teoria dei raggi beta", *Nuovo Cimento*, 11, pp. 1-19.

Heisenberg, W. (1926). "Mehrkorperproblem und Resonanz in der Quantenmechanik", *Zeitschrift für Physik*, 38, pp. 411-426.

Heisenberg, W. (1928). "Zur Theorie des Ferromagnetismus", Zeitschrift *für Physik*, 49, pp. 619-636.

Heisenberg, W. (1932a). "Über den Bau der Atomkerne I", *Zeitschrift für Physik*, 77, pp. 1-11.

Heisenberg, W. (1932b). "Über den Bau der Atomkerne II", *Zeitschrift für Physik*, 78, pp. 156-164.

Heisenberg, W. (1933). "Über den Bau der Atomkerne III", *Zeitschrift für Physik*, 80, pp. 587-596.

Heisenberg, W. (1934). "La Structure du Noyau", in *Structure et proprietes des noyaux atomiques: rapports et discussions du septième Conseil de physique tenu à Bruxelles du 22 au 29 octobre 1933*, Institut International de physique Solvay, Bruxelles, Belgium, October 22-29, 1933. Paris: Gauthier-Villars, pp. 289-323.

Heitler, W. & London, F. (1927). "Wechselwirkung neutraler Atome und höomopolare Bindung nach der Quantenmechanik", *Zeitschrift für Physik*, 44, pp. 455-472.

Ivanenko, D. (1934). "Interaction of Neutrons and Protons", *Nature*, 133, pp. 981-982.

Kawabe, R. (1991). “IV. Yukawa and Osaka-Kyoto Group”, *Progress of Theoretical Physics Supplement*, 105, pp. 241-261.
Kim, D.W. (2007). *Yoshio Nishina: Father of Modern Physics in Japan*. New York: Taylor & Francis.
Konagaya, D. (2020). “Heisenberg’s visit to Kyoto in 1929 and its stimulation of young Japanese physicists”, *Historia Scientiarum*, 29, pp. 280-299.
Konuma, M. *et al*. (2014). “The Legacy of Hideki Yukawa, Sin-Itiro Tomonaga, and Shoichi Sakata: some Aspects from their Archives”, *JPS Conference Proceedings*, 1, 013009. doi: 10.7566/JPSCP.1.013009
Majorana, E. (1933). “Uber die Kerntheorie”, *Zeitschrift fur Physik*, 82, pp. 137-145.
Massey, H.S.W. (1932a). “The Collision of α-particles with Atomic nuclei”, *Proceedings of the Royal Society of London A*, 137, pp. 447-463.
Massey, H.S.W. (1932b). “The Passage of Neutrons through Matter”, *Proceedings of the Royal Society of London A*, 138, pp. 460-469.
Miller, A.I. (1994). *Early quantum electrodynamics: a source book*. Cambridge: Cambridge University Press.
Nishina, Y., Tomonaga S. & Tamaki, H. (1936). “A Note on the Interaction of the Neutron and the Proton”, *Scientific Papers of the Institute of Physical and Chemical Research*, 30, pp. 61-69.
Oppenheimer, J.R. (1928). “On the Quantum Theory of Electronic Impacts”, *Physical Review*, 32, pp. 361-376.
Tamm, I. (1934). “Exchange Forces between Neutrons and Protons, and Fermi’s Theory”, *Nature*, 133, p. 981.
Tomonaga, S. (1946). “On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields”, *Progress of Theoretical Physics*, 1, pp. 27-42.
Tomonaga, S. (1997). *The Story of Spin*. Chicago and London: The University of Chicago Press.
Wilson, F.L. (1968). “Fermi’s Theory of Beta Decay”, *American Journal of Physics*, 36, pp. 1150-1160.
Yukawa, H. (1935). “On the Interaction of Elementary Particles. I”, *Proceedings of the Physico-Mathematical Society of Japan*, 17, pp. 48-57.
Yukawa, H. (1982). *Tabibito (The Traveller)*. Singapore: World Scientific.
Yukawa, H., Sakata, S. & Taketani, M. (1938). “On the Interaction of Elementary Particles. III”, *Proceedings of the Physico-Mathematical Society of Japan*, 20, pp. 319-340.

Archival Sources

Archive of historical materials, Osaka University Yukawa Memorial, Available at: https://www-yukawa.phys.sci.osaka-u.ac,jp/en/archive.
Tomonaga, S. (1933). Letter to Yukawa, Kyoto University, YITP, *Yukawa Hall Archive Library (YHAL) Resources*, F02 080 C01. Also available at: https://www-yukawa.phys.sci.osaka-u.ac.jp/en/archive/387.
Yukawa, H. (1933a). “The Roles of the Electron for Nuclear Structure”. Kyoto University, YITP, *Yukawa Hall Archive Library (YHAL) Resources*, E05 060 U01.
Yukawa, H. (1933b). “A Comment on the Problem of Electrons in the Nucleus”. Kyoto University, YITP, *Yukawa Hall Archive Library (YHAL) Resources*, E05 080 U01.