

Whitehead's Relational Interpretation of Special Relativity

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Abstract: The greatest relevance of Alfred North Whitehead's work, in my opinion, has to be related to the interpretation and understanding of the theory of relativity. He understood motion as a structural series of temporal events, avoiding permanent material bodies and refuting any interpretation of four-dimensional space-time as a kind of eternal permanence.

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1. The Relationality of Motion and the Relatedness of Nature

The principle of relativity, according to Whitehead, is an ontological principle, not only an epistemological one (Whitehead 1922): the impossibility of knowing the subject of motion is the consequence of the universal relatedness of Nature, an ontological principle of inter-relationship of every material body with all other material bodies, which so holistically constitute Nature. This constitutive interrelation of all material things explains why our knowledge has limitations in defining individual properties of bodies. Relations between bodies are not "ideal" relations (as in Leibniz) introduced by the human intellect to order them, but they are real: the fields of forces exist even in the absence of material bodies.

Whitehead gives a new interpretation of the "principle of (special) relativity of motion", which tells us that, in the absence of a certainly fixed reference, at rest, it is impossible to know observationally or theoretically which body, between two bodies in reciprocal motion, is at rest and which one is not at rest (in rectilinear and uniform motion), or what is the "subject of the motion".

If everything was at rest, everything would appear at rest; if a body for one reference system appears at rest and for another reference appears in motion, then either the body or the reference must be in motion: the relativity of motion implies that at least one motion exists. If the Earth is considered at rest, the Sun is moving: motion is projected on another body. In every reference frame system, there is something in motion, something changes. Even if for a body we cannot know whether it is in motion or at rest, we know that there is motion in Nature: motion as a relation is absolute, is invariant. Rest is only a relative rest, that is the situation in which two bodies have the same motion.

A transformation of reference frame transforms a motion of a body in a motion of another body (we have not the same events), but it conserves a certain temporal succession structure of events which we call motion.

The relativity of motion would not occur if there were only static reference frame systems (at rest): it is the consequence of the possibility/need to consider reference frame systems in motion.

Things do not change if we consider non-inertial motions and non-inertial reference frame systems, for which we can state a principle of general relativity of motion.

The principle of general relativity of motion tells us that it is impossible to know observationally or theoretically which body, between two bodies in reciprocal motion, is at rest and which is in motion (even accelerated in any way), or what is the "subject of motion": then or the body or the reference frame system must be in motion: the general relativity of motion implies that at least one motion accelerated in any way exists. General arbitrary transformations of reference frame systems can alter the

rest or the kind of motion of a body, but they transform an arbitrary motion of a body in the same kind of motion of another body: in different reference frame systems we have not the same events (concerning the same bodies), but a temporal succession structure of events (concerning different bodies) which identifies motion is preserved in the transformations.

In Newtonian modern physics, a body or a reference frame system is in an accelerated motion only if a force field acts on it, a field that accelerates it: experimentally, on every body or material reference frame system acts a field of gravitational forces, because gravitation is universal. Strictly speaking, all bodies – unless the gravitational field is artificially cancelled – and so all reference frame systems move in accelerated motion because they interact with all other bodies in the universe through gravitational force fields. An accelerated reference frame system, that can relativistically modify motion making it a relative thing, can exist only because there are interactions that realize a universal relatedness of Nature. There is no body or reference frame in absolute rest and a general relativity of motion is given.

The principle of general relativity of motion is the consequence of an unavoidable “solidarity of the universe”, realized through a ‘universal relatedness of Nature’, i.e., a ‘universal relationality of Nature’, a universal field of (cor-)relations. There are no isolated and separable bodies: Nature is a totality of non-separable parts.

We can have general arbitrary transformations of reference frame systems which can alter the rest or the kind of motion of a body, and which transform an arbitrary motion of a body in the same kind of motion of another body and preserve a temporal succession structure of events, concerning different bodies, only because of the universal relatedness of Nature. A change in a part of Nature must imply a change in another part (Whitehead 1920).

The principle of relativity is a principle that establishes our ignorance, an epistemological principle that concerns first of all a limit of our knowledge: in general, we cannot attribute to a single body motion as its individual property, but we can only establish it as a relationship between two bodies. We can know only in some special cases, concerning us as moving bodies, which body is moving, but motion is always a relation of a body to other bodies: for a unique existing body, we could not distinguish motion and rest. Motion is a property of Nature as a whole.

This fact, that motion for us is attributable to a body not as an individual property but only as a relationship with another body or with other bodies or relative to a certain chosen point of view (to a “frame system of reference”), leads Whitehead to conclude that in general we cannot abstract a material body from the existence of the other material bodies with which it is by nature related, i.e., that the universe is not made up of separable material bodies, but rather by bodies that cannot be separated from each other. Being in relation to other bodies constitutes the essence of a body and therefore one must consider the universe as an inter-related whole.

2. No material bodies but events. Nature as a temporal process

However, there’s more. The very concept of an individual material body separable from others loses its consistency and can no longer be the basis on which we can constitute the idea of Nature.

If we can affirm that there is a certain relationship of motion between two bodies that can never be completely eliminated (because, even when, from a certain point of view, from a certain frame system of reference, a body is at rest, we must attribute motion to another body – that is, either it is in motion one or the other), what is truly real (invariant for all the reference frame systems) is not the individual body with its supposed properties of motion or stillness that we cannot ascertain, but rather motion as a relational (collective) property of Nature. We cannot conceive a body without definite properties, it would be an abstraction. Motion is only a series of events: it is not something identifying or not a body.

Nature then is not made up of stable separable individual material bodies, but it is motion as a relation of the parts, as events: change, process.

We must include individual material bodies only as relative parts of a process, of a change-motion, which, as such, can never be described only in spatial terms, but always implies also a space-time dimension: a temporal series of events.

It then explains why in the theory of relativity we must move to a physical description in a four-dimensional space-time: because the Nature to be described is not made of individual separable, stable in some spatial position, material bodies, but rather is made of motions, changes, processes, events.

There are no more things-in-themselves-substances but only (fields of) events.

Different relations of motion between different parts of Nature imply different temporal relations. Nature is a set of different processes-motions, a set of different temporalities.

We understand that space must also be rethought in terms of time and we can also understand it in our experience if we do not make abstractions. The weft of space is woven by the vertical warp of the times.

A point in timeless space is not a fundamental entity, but it is the historical-temporal set of events, of the processes that happened there: $P = [e_1, e_2, e_3, e_4, \dots]$

This kind of conception is in close agreement also with our actual experience of space and time. For example: what is a city like Messina? Is it just a spatial place that we can know by means of geography? Is it just where we are now? That kind of definition would be reductive. Isn't it also the place where my parents, ancestors, or other people lived in the past? Isn't it also the Republic in the seventeenth century subjugated by the ferocious Spaniards? Isn't it also the place where the Turks killed and ruled? Isn't it also the place where the Greeks from Messana in Greece partly moved? Isn't it also the place where, after us, our children or other people will live? We understand that spatial geography is not enough to define Messina, but rather there is a need to add a historical-temporal dimension to define it. A place like Messina, as well as a point in space, is not a fundamental entity, but it is the historical-temporal set of events, of the processes that took place there.

What is a house? Is it just a place we live in now? No, to say that would be an abstraction. The house where I live now is also the house that belonged to my parents and will be my heir's when I die. The house is a time series of events, of processes, not a material building in urban space.

What is a sofa? Is it just where I'm sitting right now? No, it's the place where my parents sat, and where maybe others will sit after me, if it's not thrown away.

In 1903, only few years before Whitehead's solution, his scholar Bertrand Russell wrote, in the book entitled *The Principles of Mathematics*, that a relational theory of space and time should describe the principles of geometry in terms of sensible entities (Russell 1903). Russell noted that indeed right lines and planes are not such entities, whereas, on the contrary, metrical (distance) relations are. Russell went on saying that indeed there is a very complicated method, invented by Leibniz and revised by Frischouf (Couturat 1901, p. 420) and Peano (Peano 1902-03), by which only distance is fundamental, and the right line is defined from it, even if some of its properties can be introduced only by suitable axioms.

The field of a given distance is the whole space, at variance with the field of the relation that gives rise to a right line which is only such right line itself. Such a relation generating the right line, hence, at variance with the former, makes an intrinsic distinction among space points, that is a distinction that a relational theory has to avoid.

Pieri and others Peano's scholars have tried to formulate geometry starting from the fundamental concept of abstract motion, but they never create an entirely relational theory of geometry (Pieri 1899).

This kind of approach to a relational theory of geometry did not start from actual physics and involved a change in the fundamental concepts of geometry, metrical geometry concepts replacing descriptive and projective geometry ones at the foundation level (Russell 1897).

Whitehead's approach actually overcome this latter abstract (mathematical) one. However, after these works and Whitehead's answer, the relational question was almost completely hidden by the debate on general relativity, and specifically on the problem whether general relativity is actually a

relational theory of space, time, and motion. And it was also believed that this latter problem could be reduced to the technical problem of the embedding of the so-called Mach's principle within the framework of general relativity (Mach 1883; Grünbaum 1957; Sciama 1959, Sciama 1969).

Indeed, even if one accepts the historical analysis given by Gereon Wolters that Ernst Mach did not really reject relativity (Wolters 1987), and even if one accepts the pseudo-Machian formulation of general relativity given by Dennis W. Sciama and others, a relational theory of space, time and motion is a more complex task than this reformulation of general relativity, a task which was realized for special relativity by Whitehead.

It is well known that general relativity has turned upside down the hierarchy between kinematics (in some interpretation, dynamics) and geometry: the kind of geometry which enters in the construction of a physical theory is no longer given *a priori*, but it is defined by the kinematical, physical invariance group of transformations related to kinematized gravitodynamics (Barut 1989).

In this perspective, however, geometry has a foundation completely independent of physics at least at the non-metrical level, that is at the affine or projective geometrical level. It is mathematically constructed in a Platonist world of ideas, on its own specific axioms regarding abstract concepts as points, lines, etc., and only after this stage physics could individuate by a very problematic choice only the kind of metric, that is only the kind of metrical geometry to be understood and used only as a physical application of already given mathematical structures.

And even if one understands this determination of metrical geometry by physics in a more radical way as the emergence of a physical chrono-geometry as opposed to mathematical geometry, it is only the metrical structure of geometry that is physically determined.

Indeed, even if, apart from the Einstein's operational formulation, it was recognized only by Poincaré and Eddington (beyond Whitehead, of course), also special relativity can be interpreted as involving the breakdown of the hierarchy between geometry and physics. Here, the problem is the "elimination" of magnetic forces, and the definition of geometry is given by the kinematical invariance group of transformations related to partially kinematized electrodynamics (Giannetto 1994). Hence, already special relativity physics replaces *a priori* geometry with a chrono-geometry, but also in this case it is only metrical geometry which is determined by physics.

Whitehead, indeed, has solved the greatest question left by Leibniz: relationism actually implies that every concept and every structure within a physical theory must be defined in terms of relations among physical "elements"; no mathematical or logical concept or structure can be given independently from physical relations. Every other option leads to meta-physics. There is no conventionality of metric.

The fundamental concepts of physics like space and time cannot have any mathematically or logically given *a priori* structure. In Whitehead's formulation of special relativity, physics not only defines the metrical geometry, but it also defines non-metrical, descriptive or projective geometry, that is, geometry *tout court* from its "foundations". From this point of view, only Whitehead's relational chrono-geometry is an actual physical geometry, free from any logico-mathematical (Platonist or Kantian, any way idealistic) presuppositions. Let us consider, first of all, relationism in respect to the fundamental concepts of geometry.

Already in 1906 paper, Whitehead was pointing out that the simplicity of spatial points was in opposition to the relational theory of space: this requires points to be non-fundamental, complex entities.

The statement that the event-particle, which one can coordinatize by four quantities (p_1, p_2, p_3, p_4) , occupies or happens in the point (p_1, p_2, p_3) means only that the event-particle is only one of the series of event-particles which is the point. That is, point is only a series, a set of physical event-particles. Hence, a theory of space is not a theory of relations of objects, but of relations of events.

Whitehead explained that in the orthodox theory events are described by means of objects which occupy a dominant position, and so events are considered as a mere play of relations among objects. In this way, space theory becomes a theory of relations among objects instead of relations among events.

The consequence is that, for objects are not related to the becoming of events, space as relations among objects is considered as unconnected to time. However, there cannot be space without time, or time without space, or space and time without event becoming.

Thus, at variance with the major part of interpretations of relativity which speak about the spatialization of time, Whitehead obtained a complete temporalization of space, so overcoming all the philosophical criticism about that seeming feature of relativity.

Whitehead wrote in *The Principle of Relativity with applications to Physical Science*:

Position in space is merely the expression of diversity of relations to alternative time-systems. Order in space is merely the reflection into the space of one time-system of the time-orders of alternative time-systems. A plane in space expresses the quality of the locus of intersection of a moment of the time-system in question (call it 'time-system *A*') with a moment of another time-system ('time-system *B*'). The parallelism of planes in the space of time-system *A* means that these planes result from the intersections of moments *A* with moments of one other time-system *B*. A straight line in the space of time-system *A* perpendicular to the planes due to time-system *B* is the track in the space of time-system *A* of a body at rest in the space of time-system *B*. Thus, the uniform Euclidean geometry of spaces, planeness, parallelism, and perpendicularity are merely expressive of the relations to each other of alternative time-systems. The tracks which are the permanent points of the same time-system are also reckoned as parallels. Congruence – and thence, spatial measurements – is defined in terms of the properties of parallelograms and the symmetry of perpendicularity. Accordingly, position, planes, straight lines, parallelism, perpendicularity, and congruence are expressive of the mutual relations of alternative time-systems (Whitehead 1922, pp. 8-9).

Let us consider now properly kinematics. Motion is another relation of events, that is a series of events (p_1, p_2, p_3, p_4) linked to an object, conceived as placed in them, which is defined by its relation with the remaining part of the universe. If one considers another time-system (reference frame), the same motion will appear as a relation of other events (q_1, q_2, q_3, q_4), which in general are associated to other different objects (Whitehead 1922, p. 9).

Hence, even if the motion of one object is relative to the particular considered time-system, such a motion cannot be reduced to an overall rest in any other time-system: that is, it will transform itself into the motion of the remaining part of the universe. Indeed, Whitehead kinematized the concept of physical field of an object: it is nothing else than the collection of modifications of event series related to that object: it is a kinematical relation among events and it does not involve any contact or at-a-distance action (his theory of gravitation was not conceived as an action at-a-distance theory as often stated).

Thus, what is a material body in general? It is a time series of events, of processes. Nature is the process of all the interrelated processes. The visible space for us, given the finite speed of light, is not only what happens in our present, but the set of different pasts of all the other processual temporalities of all the other parts of the universe: visible space is the unfolding of different times.

3. Whitehead's Interpretation Against Some Recent Eternalist Philosophies of Special Relativity

Whitehead's interpretation of relativity can be used to refute some recent philosophies: eternalism, fourdimensionalism, perdurantism, endurantism, exdurantism. According to Whitehead, reality is, as the medieval philosophers of motion said, a *res successiva*, never a timeless object.

Four-dimensionalist philosophy proposes to consider all the temporal phases-parts of processes as coexisting simultaneously and to consider as real four-dimensional objects extended in time as well as in space, reducing time to spatial extension and thinking of such objects as persistent in time. It is a matter of considering temporal succession as illusory and time as unfolded as simultaneously: past, present and future would always coexist simultaneously in a vision linked to an eternalism opposed to

presentism that considers only the present real. You would have a block *time* or a block *universe* as an immutable four-dimensional block. The prospect of considering past, present and future coexisting simultaneously is not justifiable on the basis of the impossibility of establishing in relativity a temporal order, invariant for all reference systems, for non-causally connectable events (events linked by a *space-like space-time* interval), nor on the basis of the sole authority of Albert Einstein who has been appealed, by Federigo Enriques and Karl Raymund Popper, as the “new Parmenides” for his refusal to consider temporal succession real. The impossibility of establishing an invariant temporal order for certain events (*spacelike*) and not for all implies only the incommensurability of different temporal sequences of events at different points in space, and never a real simultaneity of all events: such different temporal sequences of different physical systems can however be compared and ordered temporally within the broader order of the temporal sequence of a system that it comprises as parts and includes all the events of the spacelike timelines in a *time-like* interval relation. The order of time is local, as in the case where, while for Galileo the life of Lucretius belonged to the past, for Epicurus it was part of the future. The order of time is also local because time is flowing. However, if we consider the history of mankind as a timeline, today we can include in its past both Galileo, Epicurus and Lucretius.

At the basis of relativity is the loss of meaning of the possibility of establishing simultaneity at a distance in space, with the consequence that a distance in space must instead be interpreted in terms of a temporal sequence of events. Eternalism is thus a logical and physical fallacy and constitutes a total misunderstanding of the processual-temporal character of four-dimensional space-time, as explained by Whitehead.

Following the four-dimensionalist philosophy, the Lorentz contraction is not real but is only a three-dimensional projection of reality: in the three dimensions there is the contraction of lengths with the dilation of time that can be real due to the magnetic field or it can be simulated by the change of the reference system; but the instantaneous section is an abstraction always because the instant does not exist and there is no simultaneity at a distance and an instantaneous space. Contraction is always related to a dilation of a time interval and therefore is not related to an instantaneous section: four-dimensionalist interpretation is thus mistaken. That the contraction and dilation then change or disappear in a particular other reference is obvious: the four-dimensional space-time volume is the same, but it indicates our ignorance about true time and true space. Space is time and varies according to the rhythm of the time of the process and the reference, space-time is not a 4-dimensional space but Whitehead showed it as a temporally characterized space; the volume therefore depends on the rhythm of time with which it is measured, but if we multiply it by time, the gamma factor is elided in $\Delta r \Delta t$ and we get the volume for the proper time of the process. If space is full of objects is given, but empty space does not exist: the empty space that exists, for example, between two celestial bodies, is only that which can be traveled or that is actually traveled. It is a different thing if one travels through it with a motion at a certain speed or with another motion at another speed. Time defines space: this definition is such that the space-time interval or the space-time product is invariant, because it is the one defined by light in a univocal way as the distance between two events or as the “evolutionary volume” of a system of certain spatial dimensions in time. Space-time as an interval indicates proper time, which we do not know what it is. What is invariant is the motion of light that corresponds to a space-time volume for which the magnetic field may or may not exist.

Four-dimensionalist eternalism has been declined in two versions: *Perdurantism*, which imagines reality as given by the mereological set of all the temporal parts that make up a single four-dimensional object that endures, referred to as a space-time worm, like the enduring set of various rings corresponding to the temporal parts; and *Exdurantism* (*Ex-durantism*), according to which persistent objects are the individual temporal parts (time-slices, instantaneous temporal slices derived from cuts in space-time) that constitute individual stages that bind together in a gen-identity relationship.

Endurantism, on the other hand, is the philosophical perspective according to which persistent objects are three-dimensional material objects that are completely present in every moment of their existence: this perspective is linked to an *A*-theory of time (in which time is thought of as a continuous transformation of events from future to present to past) by John McTaggart (1908); while *Perdurantism* is linked to a *B*-theory of time (in which events are not thought of in their flow but in tenseless relationships, i.e., without the temporal specifications of past, present and future, and therefore static-spatial of “before” and “after” that remain stable).

Both *endurantism*, *perdurantism* and *ex-durantism* postulate the persistence of objects, respectively three-dimensional or four-dimensional, whereas, according to Whitehead, the theory of relativity implies a physical reality given by a temporal processuality of non-persistent events.

The 4-dimensional space-time was introduced in 1905 by Henri Poincaré within the new relativistic electromagnetic dynamics: its introduction was necessary in the perspective in which it was demonstrated the possibility of understanding the phenomena related to material bodies in terms of phenomena of the electromagnetic field; the electromagnetic field, consisting of electromagnetic waves, is a form of motion. The movement of electromagnetic waves cannot be described only in a static spatial geometric framework but also requires time, a temporal dimension. While material bodies can be at rest at a certain moment and a three-dimensional geometry that allows them to be placed in a certain position at a certain point in space can be enough to describe their state, electromagnetic waves, being a form of wave motion, necessarily also require the temporal dimension to be described. And, therefore, the replacement of a three-dimensional geometry with a new four-dimensional “*chrono-geometry*”. The temporal dimension is thus recognized as constitutive of physical reality. Physical reality is no longer given by material bodies that can also be considered at rest, but by temporal processes (electromagnetic waves): four-dimensional space-time is the description of temporal processes.

4. Conclusions

Whitehead’s *Special Relativity* is so hierarchically structured: *Lifeworld experience* (experiments too); *Epistemology and ontology*; *Relativistic logic of events*; *Relativistic set theory of events*; *Relativistic number theory of events*; *Relativistic topology of events*; *Relativistic non-metrical chrono-geometry*; *Relativistic metrical chrono-geometry*.

Thus, Whitehead realized a relational reformulation of logic (against the metaphysics of subject-predicate logic related to the metaphysics of substances), a relational reformulation of mathematics (set theory, arithmetic, algebra, topology, non-metrical and metrical geometry), a physical reformulation of logic and mathematics.

The physics of relativity makes us understand the temporal and processual reality of things and Whitehead’s philosophy.

Whitehead’s special-relativistic theory of gravitation can be understood not as an alternative to Einstein’s general relativity theory, but in terms of a special-relativistic limit of the general relativity theory of gravitation. General relativity has been formulated as having two limits to which it reduces itself to previous theories: locally in space-time general-relativistic dynamics reduces itself to special-relativistic dynamics; and furtherly, in the limit of weak fields, general relativistic theory of gravitation reduces itself to the Newtonian theory of gravitation. Thus, the limit of general relativity is schizophrenic: Whitehead’s special-relativistic theory of gravitation filled a structural gap and made possible to consider special relativity as the unique limit of general relativity theory.

Following Whitehead, for the relationality of Nature, each part is involved in everything: one part is the set of all relations with the rest of the universe (*togetherness*): it is the relationship with all the other parts, with the otherness that constitutes every part of the universe. Nature is an inter-related totality: it

is therefore not like a machine, but constitutes a living organism. Every part of Nature is sensitive to the others, every part is alive in different degrees. A new non-mechanist image of Nature.

Process and Reality (Whitehead 1929) can be understood in terms of Whitehead's interpretation of relativity.

The relational ontology of Nature implies a cosmic relational ethics, respectful of all other parts of the universe, of every living part. One new relational image of God as a love that grows with always new relationships of the creative process of the universe.

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