DOI: 10.6093/978-88-6887-267-0

What Means that the Book of Nature is Written in Mathematical Characters?

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Abstract. In his work *Il Saggiatore*, Galileo claimed that the book of nature is written in mathematical characters. What does this statement exactly mean? It is possible to offer different answers: 1) one could think that Galileo had a strong Platonic view. This means that a mathematical world separated from the phenomenal one exists and that the physical laws are an empirical transcription of the relations existing in this word; 2) a weak Platonic interpretation is also possible: mathematics does not exist in a separated world. It is the nomological structure of our universe. It is inherent to the universe itself; 3) an operational interpretation could be considered: mathematics is the best method to measure the objects and to formulate relations among them. It is an unavoidable means for any physical research, but we cannot assume a definitive position as to the relations between mathematics and structure of the world. Simply, mathematics is the best available instrument. I will try to clarify the relation Galileo saw between mathematics and physics in the light of classical works as those by Koyré, Geymonat and Cassirer.

Keywords. Relation mathematics-physics, Galileo, Platonism, operationalism, *The Assayer*, Koyré, Geymonat, Cassirer.

1. Introduction

In his work *Il Saggiatore* (*The Assayer*), Galileo claimed the celebrate sentence that the book of nature is written in mathematical characters. What does this statement exactly mean? A huge amount of literature exists on such a question and, more generally, on the role of mathematics in Galileo's physics and on his conception of the mathematical objects. In order to provide the reader with a panoramic view of this topic and to develop some considerations, which is the purpose of this paper, it is possible to offer three different answers:

1) Galileo had a strong Platonic view. This means that a mathematical world separated from the phenomenal one exists and that the physical laws are an empirical transcription of the relations existing in this world.
2) A weak Platonic interpretation is also possible: mathematics does not exist in a separate world. It is the nomological structure of our universe. It is inherent to the universe itself.
3) An operational interpretation could be considered: mathematics is the best method to measure the objects and to formulate relations among them. It is an unavoidable means for any physical research, but we cannot assume a definitive position as to the relations between mathematics and structure of the world. Simply, mathematics is the best available instrument.

In the course of this paper, I will offer my interpretation concerning the most appropriate choice among the three previous ones as to Galileo.

2. Mathematics and *The Assayer*

One session of the SISFA Congress 2023 has been devoted to *The Assayer*. Although this text is less mathematical than other Galilean works, several considerations on the use of mathematics in physics are proposed so that it plays a significant role for our issue.

With regard to the nature of mathematics and to the relation mathematics-physics, at the beginning of *The Assayer*, we read:[1]

> 1) My Lord, the Most Serene Grand Duke Cosimo the Second, of glorious memory, commanded me to write my opinion on the reasons why things float or sink in water; and in order to comply with this commandment, in addition to the teaching of Archimedes, which is as true as anything that could be said on this subject, all the presses were immediately full of invective against my speech; Neither having any regard for the fact that what I produced was confirmed and concluded with geometrical demonstrations did they contradict my opinion, nor did they realise (so strong was their passion) that to contradict Geometry is to deny the truth outright.[2]

In the *Examen Primum*, while speaking of the possible use of parallax in the theory of comets, Galileo pronounces the sentence:

> 2) Philosophy is written in this great book that is constantly open before our eyes (I say the universe), but it cannot be understood without first learning to understand the language and to know the characters in which it is written. It is written in mathematical language, and the characters are triangles, circles, and other geometrical figures, without which it is impossible to humanly understand the word; without these it is a vain wandering in a dark labyrinth.[3]

In the same context, he writes:

> 3) So that, to say what I want to infer, in dealing with the science that can be attained by men through demonstration and human discourse, I hold that the more perfect it is, the fewer conclusions it will allow to be taught, the fewer it will demonstrate, and consequently the less it will entice, and the fewer its followers will be: but, on the contrary, the magnificence of the titles, the grandeur and numerosity of the promises, attracting the natural curiosity of men and keeping them perpetually entangled in fallacies and chimeras, without ever giving them a taste of the sharpness of a single demonstration [...].[4]

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[1] I give a number to the following quotations because, when I will recall them, a series number is useful for the reader's orientation.

[2] "Imposemi il Serenissimo Gran Duca Cosimo Secondo, di gloriosa memoria, mio Signore, ch'io scrivessi il mio parere della cagioni del galleggiare o affondarsi le cose nell'acqua; e per sodisfar a così fatto comandamento quanto m'era sovvenuto oltre alla dottrina d'Archimede, che per avventura è quanto di vero in effetto circa siffatta materia poteva dirsi, eccoti subito piene tutte le stamperie d'invettive contro il mio *Discorso*; né avendo punto riguardo che quanto da me fu prodotto fusse confermato e conchiuso con geometriche dimostrazioni contradissero al mio parere, né s'avvidero (tanto ebbe forza la passione) che'l contradire alla Geometria è un negare scopertamente la verità" (Galilei EN, VI, pp. 213-214).

[3] "La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto dinanzi agli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezzi è impossibile intenderne umanamente parola; senza questi è un aggirarsi vanamente in un oscuro laberinto" (Galilei EN, VI, p. 232. p. 147).

[4] "Sì che, per dir quel ch'io voglio inferire, trattando della scienza che per via di dimostrazione e di discorso umano si può da gli uomini conseguire, io tengo per fermo che quanto più essa parteciperà di perfezzione, tanto minor numero di conclusioni permetterà d'insegnare, tanto minor numero ne dimostrerà, ed in conseguenza tanto meno alletterà, e tanto minore sarà il numero dei suoi seguaci: ma, per l'opposito la magnificenza de' titoli, la grandezza e numerosità delle promesse, attraendo la natural curiosità degli uomini e tenendoli perpetuamente ravvolti in fallacie e chimere, senza mai dar loro gustar l'acutezza d'una sola dimostrazione […]" (Galilei EN, VI, p. 237).

At the beginning of the *Examen Secundum*, Sarsi (Grassi) claims that Galileo considers the comet to be produced by vapours rising from the earth and illuminated by the Sun's rays. Actually, Guiducci (Galileo) only claimed that this phenomenon is possible and that it could generate the comet, not that the comet is certainly produced in this way. In this context Galileo writes:

> 4) Let this be said, not in order to retreat, for fear of being challenged by Sarsi, but only so that it may be seen that we do not deviate from our custom, which is not to affirm as certain except those things that we know indubitably, which is how our philosophy and mathematics teach us.[5]

Sarsi claims that, if the comet would be an effect of the solar rays, its movement should show any connection with that of the Sun. This is not the case. Thence, the Sun has no relation with the comet. Galileo accuses Sarsi to have used geometry incorrectly and writes:

> 5) But to reduce oneself to geometrical demonstrations is too dangerous a challenge for those who do not know how to handle them well; for just as there is no middle ground between the true and the false *ex parte rei*, so in the necessary demonstrations one either indubitably concludes or inexcusably paralogizes, without leaving room for limitations, distinctions, misinterpretation of words or other twists and turns to support oneself more, but it is necessary to keep things brief and to remain either Caesar or nothing at the first assault. This geometrical narrowness will make it possible for me, with brevity and with less tedium for Your Illustriousness, to detangle myself from the following proofs.[6]

The next reference to the necessity of using a demonstrative method in science concerns the form of the comet's tail: Sarsi, sharing Kepler's opinion, thinks the tail to be curved, whereas Galileo claims that it is straight, but appears curved. In the attempt to prove his thesis, he writes:

> 6) Very different is therefore the reason produced and rejected by Kepler; who, as a person always known to me to be no less free and sincere than intelligent and learned, I am sure that he would confess that our saying is entirely different from his own and that, just as his deserved rejection, this deserves assent, because it is true and demonstrative, even if Sarsi contrives to reject it.[7]

While discussing the form of the celestial bodies and of the skies, Galileo ascribes no perfection or imperfection to the geometrical figures. Such attributes are used improperly when referred to mathematical objects. This marks a great difference with the tradition and with Kepler himself.

> 7) And before I proceed any further, I would like to reply to Sarsi that it is not I who wants the heavens, as a most noble body, to have a most noble figure, such as the perfect sphere, but Aristotle himself, against whom Mr. Mario *ad hominem* argues. And I, for my part, never having read the chronicles and particular nobilities, do not know which of them are more or less noble, more or less

[5] "E questo sia detto non per ritirarci, per paura che ci facciano l'oppugnazioni del Sarsi, ma solo perché si vegga che noi non ci allontaniamo dal nostro costume, ch'è di non affermar per certe se non le cose che noi sappiamo indubitatamente, ch'è così c'insegna la nostra filosofia e le nostre matematiche" (Galilei EV, VI, pp. 278-279, p. 240).

[6] "[…] ma quel ridursi alle geometriche dimostrazioni è troppo pericoloso cimento per chi non le sa ben maneggiare; imperocchè, sì come *ex parte rei* non si dà mezo tra il vero e 'l falso, così nelle dimostrazioni necessarie o indubitabilmente si conclude o inescusabilmente si paralogiza, senza lasciarsi campo di poter con limitazioni, con distinzioni, con istorcimenti di parole o con altre girandole sostenersi più in piede, ma è forza in brevi parole ed al primo assalto restare o Cesare o niente. Questa geometrica strettezza farà ch'io con brevità e con minor tedio di V. S. illustrissima mi potrò dalle seguenti prove distrigare" (Galilei EN, VI, p. 296, p. 274).

[7] "Differentessima è dunque la ragione prodotta e rifiutata poi dal Kepplero; il quale, come persona conosciuta da me sempre per non men libera e sincera che intelligente e dotta, sono sicuro ch'ei confesserebbe, il nostro detto esser in tutto diverso dal suo e che, come il suo meritò rifiuto, questo merita l'assenso, perch'è vero e dimostrativo, ben che il Sarsi s'ingegni di rifiutarlo" (Galilei EN, VI, p. 313, p. 309).

> perfect; but I believe that they are all ancient and noble in one way, or, to put it better, that as far as they are concerned they are neither noble nor perfect, nor ignoble and imperfect, except insofar as for walling I believe the square ones are more perfect than the spherical ones, but for rolling and driving chariots I consider the round ones more perfect than the triangular ones.[8]

The previous quotations clarify what roles Galileo attributes to mathematics in *The Assayer*: First of all, mathematics is a criterion of truth and certainty. When it is possible to reduce physics to mathematics, no mistake can occur. Quotation 1): "[…] what I produced was confirmed and concluded with geometrical demonstrations [...] to contradict Geometry is to deny the truth". Quotation 3): while dealing with science, it is necessary to resort to "demonstration and human discourse", otherwise we are "entangled in fallacies and chimeras". Quotation 4): it is necessary to consider certain only "[…] those things that we know indubitably, which is how our philosophy and mathematics teach us". Quotation 6): it is necessary to accept what proved mathematically "[…] because it is true and demonstrative".

All the previous statements leave no doubt: first of all, Galileo regards mathematics as an essential instrument in science: as soon as we can measure and are able to use geometry, we are allowed to consider our conclusions to be certain and unquestionable. Mathematics is necessary because the measure-operations and the reasoning through which new science develops its research on physical phenomena need precise, quantified characterizations.

This has nothing to do either with the existence of a mathematical world separate from the phenomenal one, or with a mathematical structure inherent to reality. No a priori conception influenced Galileo. Rather, the fundamental element, which makes mathematics unavoidable for physics is the rigour of the results obtained by scientists such as Archimedes who used mathematics in contrast with the uncertainty and ambiguity of the results achieved by means of mere qualitative statements.

Galileo explicitly claimed (quotation 7): 1) mathematics has nothing to do with subjective attributes such as perfection and imperfection; 2) a form F is not perfect in itself, but only in relation to the utilization we have to do of an object having the form F. This strictly connects mathematics with its practical use. In some circumstances, for example to construct an edifice, a cubic form is more perfect than a spherical one (let us think of the bricks). In other circumstances, the opposite is true. Therefore, according to Galileo, mathematics enters science and technique from below and not from above. This is the fundamental aspect for grasping the conception he held of this discipline and for understanding the relationship between mathematics and science in Galileo. Therefore, the question of the ontological status of mathematical objects is a secondary question because it has a negligible influence on Galileo's science.

Nonetheless, although with this crucial limitation, such a question makes sense. Thence, one can wonder whether Galileo believed in the existence of a mathematical world separated from the phenomenal one or if he believed mathematical laws to be inherent in nature, though denying a mathematical hyperuranium. The former is a strong form of Platonism; the latter a weak one.

In this regard, the assertion in *The Assayer* that the world is written in mathematical characters has been interpreted as a clear indication of Galileo's Platonism, at least in its weak form. However, this could be an overreading because what Galileo states is that nature can be deciphered only if we know the language of mathematics. How mathematics can be applied to nature is a question that cannot be solved a priori. Experiments and observations are necessary.

[8] "E prima che più avanti io proceda, torno a replicare al Sarsi, che non son io che voglia che il cielo, come corpo nobilissimo, abbia ancora figura nobilissima, qual'è la sferica perfetta, ma l'istesso Aristotile, contro al quale si argomenta dal Sig. Mario *ad hominem*; ed io, quanto a me, non avendo mai lette le croniche e le nobiltà particolari, non so quali di esse sieno più o men nobili, più o men perfette; ma credo che tutte sieno antiche e nobili a un modo, o, per dir meglio, che quanto a loro non sieno né nobili né perfette, né ignobili ed imperfette, se non in quanto per murare credo le quadre sieno più perfette che le sferiche, ma per ruzzolare e condurre i carri stimo più perfette le tonde che le triangolari" (Galilei EN, VI, p. 319, p. 325).

3. A hint on the literature concerning Galileo's conception of mathematics

Many authors dealt with the problems of understanding the role ascribed by Galileo to mathematics within physics and his conception of mathematical objects. In this paper, it is impossible to enter many details and nuances of such a complicated topic, but it is possible and useful to recall the most important interpretations, although in their general terms.

Alexandre Koyré is the most important interpreter who considers Galileo as a Platonist. In his *Études Galiléennes* we read: "In front of abstractive empiricism, Galileo vindicates the superior right of Platonic mathematism" (Koyré 1966, p. 78). In his essay "Galileo and Plato" (Koyré 1943), the author regards the mentioned passage of *The Assayer* on the book of nature written in mathematical characters as a proof of his thesis. He also analyses some passages of the *Dialogo*'s Third Day, where Galilei clarifies that a mathematical argumentation must be preferred to any other kind of other reasoning. Koyré stresses that experiments were secondary in Galileo's science: the mathematical aspect was preponderant. Galileo was always intent on investigating the ultimate structure of Being rather than on solving single physical problems. Without entering the complex question of Galileo's experimentalism and its link with mathematization,[9] it is sure that Galileo used mathematics exactly to solve single problems - a long series of single problems -, and not to investigate on the ultimate foundation of Being. In this respect, he is like a modern physicist: mathematics is used because it is the most precise language usable in physics, independently of the concept one can hold as to the nature of mathematical objects and as to the relations physics-mathematics. The structure of his major works such as the *Dialogue Concerning the Two Chief World Systems,* the *Dialogues Concerning Two New Sciences* and *The Assayer* itself, leaves no doubt about this topic. Koyré recognizes that Galileo did not believe in the existence of an ideal mathematical world separate from the physical one. Thus, he was simply not a Platonist, or, if one prefers, he was a weak Platonist. But this is only a terminological question. Instead, it is important that the use of mathematics in Galileo – which is the fundamental aspect – is not related either to his conception of mathematical objects or to the links he believed to exist between mathematics and physics. This means that Galileo understood mathematics to be a necessary instrument in physics and that every physicist has to use such an instrument, independently of his philosophical views on mathematical entities and on the relations physics-mathematics. Be the mathematical laws inherent to the universe or be they only an approximation of reality, they cannot be avoided in physics. We told that this way to introduce mathematics within physics is "from below". In contrast to this, Koyré believes that Galileo's philosophical convictions played a significant role. Namely, according to this scholar, Galileo introduced mathematics in physics "from above".

Ludovico Geymonat has a different opinion from Koyrè's: for Galileo mathematics does not represent either a reality in itself or an investigative instrument superior to experience, but an unavoidable means to interpret experience itself. Galileo, through a synthesis of experience and mathematics, invented the modern scientific method based on; (1) the rejection of hypotheses which cannot be verified; (2) the verification and confirmation of a theory by successive steps (Geymonat 1981, pp. 119-ff). Although Geymonat's interpretation of how Galileo considered and used mathematics is more consonant to the one I propose, he reads Galileo as a too systematic and methodic thinker. This is not the case. In this respect, Feyerabend's view is sharable. Let us refer, e.g., to Galileo's use of the telescope. For his astronomical observations he resorted to an instrument, the telescope, whose optical theory was not yet known. Through this instrument he tried to validate the truth of an astronomical system (the Copernican one) that was not supported by sufficient observations (in particular, because of the lack of any detectable stellar parallax). This is certainly not the way of proceeding of a systematic thinker and experimenter (Feyerabend 1975, chapters 8-10). Galileo himself is clear on this aspect because in his *Capitolo contro il portar la toga* (*Against the Donning of the Gown,* written around 1590.

[9] On this aspect, see, e.g., Bussotti-Lotti 2022, pp. 102-106. We also offer abundant references to the literature.

See: Galilei 2009) he claimed that no a priori method exists in scientific research: if the usual procedures are useless for solving a problem, the scientist has to work with art and invention, since thousands of other ways can be exploited to reach the truth (see Bussotti-Lotti 2022, p. 108).

Ernst Cassirer expressed a position which is different from both the previous two and which catches several important aspects of our question: Galileo is a Platonist in a weak meaning of this term. He developed a new conception of truth: a profound interconnection exists between mathematics and nature. Mathematics represents, so to say, the immanent essence of the world. However, the idea of the mathematical character of physical truth is, in Galileo, a subjective preconception. It is not connected to any philosophical school (Cassirer 1922, pp. 383-ff). Most aspects of Cassirer' ideas can be shared. The weak element of his thesis is the difficulty to prove if Galileo's statement that mathematics is inherent to nature was a subjective preconception or a belief developed during his studies when Galileo understood that, without quantifying the magnitudes, a satisfying solution to the problems of physics was impossible. This latter option seems preferable.

4. Conclusion

Let us now summarize the ideas here proposed:

1) According to Galileo, mathematics is indispensable to physics as it is the only instrument to measure exactly any magnitude. Therefore, the first and essential use of mathematics is the operative one.
2) No mathematical hyperuranium exists, but physical laws mathematically expressed do exist in nature. He was a "weak Platonist". In any case, Galileo's achievements, obtained by means of the use of mathematics in his physical investigations, do not depend on a philosophical view about the nature of mathematical objects or about the relationship between mathematics and physics.
3) He did not believe either in anything like "mathematical archetypes inscribed in the universe" à la Kepler, or in a transcendent world of mathematical entities separate from physical reality and its functioning laws. He was not, therefore, a Platonist in the classical sense of the term.
4) He had a clear idea of the importance of experiments for physical investigation. Nonetheless, he did not adopt a scientific method in which experiments, and mathematics are coordinated in a precise and systematic a priori way.

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