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Dirac and Quantum Times

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Abstract: In 1926, Dirac attempted to give a relativistic generalization of his mechanics of q-numbers, defining a new "quantum time" starting from the relation: $tW - Wt = -ih$. In the 1927 article, Heisenberg starts from this relationship by Pauli and Dirac which implies this new concept of quantum (relativistic) time. All the historical formulations of quantum mechanics were time irreversible. Time was a matrix in Heisenberg matrix mechanics and all the physical variables obeyed a matrix evolution, time irreversible equation. Schrödinger equation was written as representing the propagation of an irreversible physical wave. One has been able to state that quantum mechanics is time reversible only by defining the time reversal operator in a new way, involving also complex conjugation, just to obtain reversibility. This new formal insight was obtained for the first time only in 1932, in the paper *Über die Operation Zeitumkehr der in der Quantenmechanik* by Eugene Wigner. He introduced some sort of mathematical trick by defining the time reversal operator in a new way involving complex conjugation. Irreversibility of quantum mechanics is shown by giving a new insight into quantum time and motion.

Keywords: Quantum Time, Time Reversal Operator, Irreversibility; Quantum Mechanics

1. The Introduction of a *Quantum Time*

After the revolution of special relativity and general relativity, there was an unknown revolution in the concept of time also in quantum physics. It occurred very early, but was then denied and finally sent back to quantum gravity. At this level, it was then further denied, based on a classical idea of time. Quantum time is a discrete, discontinuous, multiple, irreversible time, linked to all the variety of processes and physical quantities that characterize them. It must not be identified with the idea of the existence of a minimum time, the so-called chronon (Kragh & Carazza, 1994). The concept of quantum time dates back at least to a work by Henri Poincaré (1854-1912) published in 1912 in which he presented an early form of quantum mechanics (Poincaré, 1912a, 1912b; Giannetto, 2005). The deepest recent search for a fundamental quantum time was done by David Finkelstein. It was through Dirac's work that in 1926 the concept of quantum time finally entered physics (Dirac, 1926). Dirac not only attempted to create a new mechanics as a synthesis of wave mechanics and matrix mechanics, but even before that Dirac had attempted to give a relativistic generalization of his q-number mechanics, defining a new quantum time starting from the relation: $tW - Wt = -ih$, where W indicates the energy (Giannetto, 2005).

Quantum numbers, *q-numbers*, are indeed ordinal numbers, temporal numbers: they do not commutate because of the temporal order: they indicate temporal irreversibility of measurement operations and general, temporal, physical processes.

Dirac, in his theory of transformations, had pointed out how the Schrödinger wave function corresponded to nothing other than a transformation from a scheme with diagonal position to one with diagonal energy in the language of matrices. That the system is described by a certain wave function means considering it as a process that can never be stationary in position energy or momentum or any other variable, except within the limits of a wave characterization.

This implied to Heisenberg that the fundamental concept of wave mechanics could be traced back to matrix mechanics. Matrices described temporal processes, thus wave functions did not describe states of a system, but temporal wave processes of some physical variable. Matrices, on the other side, were defined by Heisenberg in terms of a finite sum of Fourier analysis, which allows a decomposition of every physical variable change in terms of elementary sinusoidal and co-sinusoidal waves. Heisenberg considered the breakdown of the solar system model of the atom since Max Born has shown that an $N = 2$ atom presented an unsolvable three-body problem (Born, 1924). Thus, Heisenberg thought to describe electron motions as planet motions in Babylon algebraic astronomy, where the problem was the irregularity of trajectories, or the non-existence at all of a definite trajectory because the planets have many retrograde motions: Heisenberg introduced ephemeris-like (sinusoidal) time tables (Heisenberg, 1925) for electron motions, then interpreted as matrices (Born & Jordan, 1925; Born, Heisenberg & Jordan, 1926).

The problem of the concept of time played a very important role in Heisenberg's uncertainty relations.

Pauli had already hinted at the idea of defining time in terms of energy in a letter to Bohr dated 17 November 1925 (Pauli, 1979, vol. I, pp. 257-261). In the 1926 encyclopedia article, he highlighted how energy transitions were not describable according to exact times, as there was an imprecision in the moment of the transition and a problem regarding its duration.

In response to a letter from Heisenberg dated 27 January 1926, Pauli, in a letter dated 31 January 1926 (Pauli, 1979 vol. I, pp. 281-288), gives a new physical definition of time through the relationship, in a matrix formalism,

$$Wt - tW = \frac{1h}{2\pi i} \tag{1.1}$$

where W indicates the energy.

The concept of quantum time was therefore already born in the context of a synthesis between quantum mechanics and special relativity: if the position is a q-number, the Lorentz transformations imply that time is also a q-number.

In the 1927 article, Heisenberg starts from this relationship by Pauli and Dirac which implies this new concept of quantum (relativistic) time (Heisenberg, 1927). Even if Heisenberg's indeterminacy relations do not contain all the relativistic implications, which make them even more limiting, they arise within a relativistic consideration.

Quantum-relativistic complete implications indeed amount to the fact that it is no longer possible to obtain measurements relating to momentum and energy in an instant or in a short time since their accuracy would require an infinite amount of time (as regards momentum, the indeterminacy is around $\frac{\hbar}{c}$ for the inverse of temporal indeterminacy, $\Delta p \Delta t \sim \frac{\hbar}{c}$. The error on the position of a particle can end up coinciding with the De Broglie wavelength, $\Delta x \sim \frac{\hbar}{mc}$ or $\frac{\hbar}{p}$, which implies that the photon is never localizable) (Landau & Lifshitz, 1958).

It follows that uncertainty relations cannot be formulated in terms of simultaneous measurements of conjugated variables, as simultaneity is relativistically dependent on the reference system and can be rigorously defined only for events that occur at the same spatial point. The wave function of a certain size at a given instant is not definable, it does not correspond to observable and physically available information: it must be interpreted in terms of a space-time process, that is of an actual (ontological) finite, discontinuous multiplicity of quantum physical wave fields. The indeterminacies on the momentum and energies correspond to those of possible processes of creation and destruction of a particle-antiparticle pair $\Delta p \sim mc$ and $\Delta E \sim mc^2$, which means that the possibility of attributing them to a specific material particle is no longer valid, which means that particles can no longer be considered isolated and localizable objects, but waves.

In Heisenberg's 1927 formulation, which took Dirac's quantum time into account, the two indeterminacy relations, $\Delta q \Delta p \sim \hbar$ and $\Delta E \Delta t \sim \hbar$ were interpreted symmetrically. The Δt was not interpreted as a simple duration of the process but as an effective error on a quantum variable that can only be known probabilistically (Heisenberg, 1927).

In the letter to Pauli dated November 23, 1926 (Pauli, 1979, vol. I, pp. 357-360), Heisenberg wrote that "it is completely impossible for the world to be continuous" and "what the word 'wave' or 'particle' means is no longer known". And he added: "if space-time is discontinuous, the velocity in a defined point cannot have any meaning, because, to define the velocity in a point, there is a need for a second point infinitely close to the first: that is impossible in a discontinuous world".

Mathematically, Heisenberg's reflection can be translated as follows: $v = \frac{ds}{dt}$ and this also makes the definition of the momentum impossible, and being $E = h\frac{\omega}{2\pi} = \frac{d\theta}{dt} \cdot \frac{h}{2\pi}$ that is, since the pulsation ω is associated with an angular velocity, even the definition of energy becomes impossible if space-time is discontinuous. It is not possible to calculate the limit of the incremental ratio that defines the speed: time intervals, such as spatial or angular ones, cannot tend to zero, and mathematical discontinuities correspond to finite and irreducible experimental indeterminacies of physical quantities. Quantum time means discontinuous time, linked to discontinuous physical processes of change. Indeterminacy and discontinuity imply irreversibility. Usually, however, this irreversibility has been confined to the act of measurement. In classical mechanics, motion is considered reversible and so is time, as a spatial trajectory can be traveled in both directions. In quantum mechanics, it is not possible to trace a motion trajectory linked to a motion time law that gives the position as a continuous function of time. In the matrix mechanics of Heisenberg, Bohr, and Jordan, time is a matrix, and the matrices representing the physical processes of change (transition) are linked to temporally irreversible equations.

In any case, since Dirac's, Pauli's, and Heisenberg's works the concept of quantum time changed the concept of time: while, before them, it was connected to space and therefore to the process which, from Aristotle onwards, is called local motion, Heisenberg, under the suggestion of Pauli and Dirac, with the relations of indeterminacy, defines quantum time by connecting it to more general energetic processes of energy change. And here an irreversible, discontinuous, quantum time emerges. It was not discontinuity, discreteness of time to be not accepted. Now, the concept of an irreversible quantum time encountered very strong opposition when one wanted to consider quantum mechanics as temporally reversible like classical mechanics. Reversibility prepares a return to a time that can only be classical, like a simple mathematical parameter, to which no reality of irreversible change corresponds.

From a Heraclitean vision in which time was the constitutive foundation of *Physis*, we returned to a Parmenidean timeless vision also in quantum physics (Prigogine & Stengers, 1979).

2. The transformation of the time reversal operator

Quantum mechanics is usually considered as time reversible (Messiah, 1961, vol. II, pp. 664-675; Landau & Lifshitz, 1958), even if sometimes this "axiom" has been challenged (Racah, 1937; Schrödinger, 1950; Watanabe, 1955, 1965; Jauch & Rohrlich, 1955, pp. 88-96; Costa de Beauregard, 1980; Healey, 1981; Callender, 2000). Schrödinger's wave mechanics is considered as time-reversible and the other (Heisenberg's or Dirac's) formulations of quantum mechanics are considered as equivalent to wave mechanics (Giannetto, 1999). Quantum time asymmetry is so usually limited to the act of measurement (Reichenbach, 1956; Davies, 1974; Sachs, 1987; Zeh, 1989). However, it is not clear at all that this property of quantum mechanics is a theoretical or better an epistemic postulate. The deep reason for this postulate is related to the continuity (analogy) with classical mechanics. Quantum mechanics should be time-reversible because classical mechanics is time-reversible. In classical mechanics, motion should be time-reversible because a spatial trajectory is time-reversible.

However, all the historical formulations of quantum mechanics were time irreversible. Time was a matrix in Heisenberg matrix mechanics and all the physical variables obeyed a matrix evolution, time irreversible equation. Schrödinger equation was written as representing the propagation of an irreversible physical wave (Giannetto, 1999).

One has been able to state that quantum mechanics is time reversible only by defining the time reversal operator in a new way, involving also complex conjugation, just to obtain reversibility. This new formal insight was obtained for the first time only in 1932, in the paper *Über die Operation Zeitumkehr der in der Quantenmechanik* by Eugene Wigner. It must be remembered that the time reversal operator must be defined in a new way to yield a time-symmetrical quantum mechanics. This was realized for the first time by Eugene Wigner (1931, 1932). Time reversal τ is defined an antiunitary operator, associating the transformation θ of t into $-t$ with a conjugation operation $*$ changing i into $-i$, in such a way that

$$\tau = \theta^* \tag{2.1}$$

$$\tau[\psi(x,t)] = \psi^*(x,-t) \tag{2.2}$$

where ψ is the wave function of a quantum mechanical system.

This definition is such that Schrödinger equation

$$H\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t} \tag{2.3}$$

where H, the Hamiltonian operator, is invariant under τ transformation.

For a free particle

$$H = \frac{p^2}{2m} = \frac{\hbar^2}{2m}\nabla^2 \tag{2.4}$$

and

$$p = -i\hbar\nabla \tag{2.5}$$

is invariant under τ transformation and the change in i is counterbalanced by the change of sign in t and the temporal partial derivative. This corresponds to the requirement that momentum changes its sign and energy does not change under time reversal as in classical mechanics and the canonical commutator between x and p is invariant: $[x,p] = i\hbar$.

However, these requirements are related to the classical concepts of momentum and energy and one cannot neglect that within quantum mechanics momentum and energy are no more classical variables but are associated with operators, which has no intrinsic physical meaning.

Within quantum mechanics, in the usual representation of eq. 2.5 momentum does no more depend on time, and so it does not change by changing t into $-t$. And even if one does not associate energy to an operator $i\hbar\frac{\partial}{\partial t}$, Schrdinger equation is not invariant by changing t into $-t$ by transformation θ and makes energies change into negative values.

From

$$\theta[H\psi(x,t)] = \theta[i\hbar\frac{\partial\psi(x,t)}{\partial t}] \tag{2.6}$$

it follows:

$$H\psi(x,t) = -i\hbar\frac{\partial\psi(x,-t)}{\partial t} \tag{2.7}$$

that is, $\psi(x, -t)$ does not obey the same equation.

However, this result cannot be a reason to change the definition of time reversal operator into an antiunitary operator. On the contrary, this must be taken as proof that quantum mechanics is not invariant under time reversal, just because a time inversion would change energies to negative values. Evolution operator $e^{-iH\frac{t}{\hbar}}$ under time reversal would imply negative energy values. For a θ time reversal transformation there is no probability (P) conservation and so no information conservation: $\frac{\partial P(x,-t)}{\partial t} = \nabla J(x, -t)$. Thus, time inversion cannot have any physical meaning.

This does not allow any dualism between a time-reversible, quantum dynamics and an irreversible quantum measurement.

3. Pauli against Dirac's Quantum Time

The need for classical time, as a parameter and not as a quantum operator, was due to the maintenance of temporally reversible quantum mechanics.

The main argument against quantum time is paradoxically due to Wolfgang Pauli:

> In the older literature on quantum mechanics, we often find the operator equation $Ht - tH = \frac{\hbar}{i}I$
> It is generally not possible, however, to construct a Hermitian operator (e.g. as a function of P and Q) which satisfies this equation.
> This is so because, from the C.R. written above, it follows that H possesses all eigenvalues from $-\infty$ to ∞ whereas on the other hand, discrete eigenvalues of H can be present. We therefore conclude that the introduction of an operator t is basically forbidden and the time t must necessarily be considered an ordinary number ("c-number") in Quantum Mechanics. (Pauli, 1980, p. 63)

Pauli's objection to the possibility of a time operator was linked to the fact that since energy is classically the generator of (continuous) time translations, every time operator must be conjugated to an energy operator (Hamiltonian) that has a spectrum unlimited continuum, a property which is not satisfied by the Hamiltonian of typical systems and which also leads to negative energy values.

However, Pauli presupposed here the continuity of time and the connection to energy. Paradoxically, the attempt to avoid quantum time linked to energy opens the way to a new concept of a multiplicity of quantum times linked to the rates of variation of each physical quantity.

4. A Plurality of Quantum Times

One has to consider also Dirac's Heisenberg operator equation for every operator A:

$$[A, H]_- = i\hbar\frac{\partial A}{\partial t} \tag{4.1}$$

that is not invariant for θ time reversal:

$$\theta[A, H]_- = \theta[i\hbar\frac{\partial A}{\partial t}] \tag{4.2}$$

implies

$$[A(-t), H]_- = -i\hbar\frac{\partial A(-t)}{\partial t} \tag{4.3}$$

Indeed, indeterminacy relations take the form:

$$\langle\Delta A\rangle\langle\Delta H\rangle \geq \frac{1}{2}\left|[A, H]_-\right| \tag{4.4}$$

so that

$$\langle \Delta A \rangle \langle \Delta H \rangle \geq \frac{1}{2} \left| [A, B]_- \right| \tag{4.5}$$

and

$$\langle \Delta A \rangle \langle \Delta H \rangle \geq \frac{\hbar}{2} \frac{\partial A}{\partial t} \tag{4.6}$$

In general, indeterminacy relations define a new concept of quantum time as a function of quantum operators: it can be associated with any change of physical variable that is irreversible by the Heisenberg equation. There is a multiplicity of quantum times (operators): there is no single temporal order. It is no longer necessary or possible to define time in terms of uniform spatial variation, but there is a different type of quantum time internal (proper) to each type of physical process. Due to the indefinability of a trajectory, local motion is also quantum unpredictable and an irreversible process (it is not possible to define a state of motion, i.e. motion as a state).

Therefore, we have that the energy-time indeterminacy relations are:

$$\langle \Delta E \rangle \langle \Delta t \rangle = \frac{\langle \Delta E \rangle \langle \Delta A \rangle}{\frac{\partial A}{\partial t}} \geq \frac{\hbar}{2} \tag{4.7}$$

and the quantum time for each A is defined:

$$\tau_A = \frac{\partial A}{\partial t} \tag{4.8}$$

Thus, if we carry out a time reversal transformation θ, we obtain

$$\langle \Delta E \rangle \langle \Delta t \rangle = \langle \Delta E \rangle \langle \Delta A(-t) \rangle > \frac{\partial A(-t)}{\partial t} \leq \frac{\hbar}{2} \tag{4.9}$$

which cannot be accepted because there is a violation of indeterminacy relations.

In this new definition of a multiplicity of quantum times linked to the variations of physical quantities, the time derivative is calculated in relation to a classical macroscopic time which acts as a reference sample. The quantum time of Heisenberg's uncertainty relations is therefore no longer directly connected to energy processes, but intrinsically to all physical variations in their discontinuity and irreversibility and is a further generalization of the concept of time. The attempt to maintain a reversible and continuous classical time has meant that the issue of a quantum time has been postponed until quantum gravity where it is unavoidable for space-time quantization, and here too the obsession with the classical concept of time does not allow it to emerge: it is said that time cannot be defined. However, if one accepts the rigorous original formulation of quantum mechanics, one has to deal with a plurality of discontinuous, discrete, finite, and irreversible quantum times.

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