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A study on Curie's paper and the many versions of "Curie's principle"

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Abstract: Pierre Curie is credited to have first introduced in 1894 symmetries into theoretical physics.. But for a long time the theoretical contents of Curie's paper remained obscure. In past decades some scholars interpreted them through a crucial proposition called "Curie's principle". But too many versions of it resulted. A previous paper showed that the word "symmetry" is a double negation without a corresponding affirmative word; that means the failure of the double negation law; hence, this word belongs to intuitionist logic and introduces the theoretical organization, the problem-based one, which is alternative to the deductive one. In the light of these novelties, I examine all theoretical propositions of Curie's paper; their contents are elucidated and their logical, mathematical and physical differences are examined in order to establish their relations. Only two of them are recognized as the correct ones for recognizing the symmetry of a phenomenon inside a medium: a mathematical inequality and a group theoretical formula; both belong to intuitionist logic like the word "symmetry". The remaining propositions do not circumscribe the subject. A similar analysis is performed on the interpretative versions of Curie's principle suggested by several scholars. In order to avoid interpretative difficulties, someone bound the theoretical framework of it to be deterministic and temporal. The remaining others chose the same two above propositions as representing Curie's principle, yet without explaining the remaining theoretical propositions. Few of these scholars closely approached a well-defined version of the principle and so put the basis for building a theory of Curie's paper according to a problem-based theory. In conclusion, there exists a theoretical proposition which can be called the "Curie principle" and has to be applied as a heuristic principle within a problem-based theoretical organization.

Keywords: Pierre Curie, Curie's principle, symmetry as a double negation, intuitionist logic, problem-based organization

1. Introduction

Pierre Curie is credited with introducing in 1894 symmetries of finite systems into theoretical physics. He wanted to discover which physical symmetrical phenomena are allowed to happen within a given physical medium (e.g. a crystal) having specified symmetry properties[1] .

[1] To help the reader I quote, as a provisory summary of Curie's paper, what recently three scholars suggested about the conclusions of this paper and its theoretical importance: "a. A phenomenon can exist in a medium possessing its characteristic symmetry or that of one of its subgroups. What is needed for its occurrence (i.e. for something rather than nothing to happen) is not the presence, but rather the absence, of certain symmetries: *Asymmetry is what creates a phenomenon*./ b. The symmetry elements of the causes must be found in their effects, but the converse is not true; that is, the effects can be more symmetric than the causes./ Conclusion (a) clearly indicates that Curie recognized the important function played by the concept of symmetry breaking in physics (he was indeed one of the first to recognize it). Conclusion (b) is what is usually called *Curie's principle* in the literature, although notice that (a) and (b) are not independent of one another./ In order for Curie's principle to be applicable, various conditions need to be satisfied: the causal connection must be valid, the cause and effect must be well-defined, and the symmetries of both the cause and the effect must also be well-defined (this involves both the physical and the geometrical properties of the physical systems considered). / Curie's principle then furnishes a necessary condition for given phenomena to happen: only those phenomena can happen that are compatible with the symmetry conditions established by the principle. Curie's principle has thus an important methodological function: on the one side, it furnishes a kind of selection rule (given an initial situation with a specified symmetry, only certain phenomena are allowed to happen); on the other side, it offers a falsification criterion for physical theories (a violation of Curie's principle may indicate that something is wrong in the physical description)" (Brading *et al.*, 2023, sect. 3).

But after a century and more scholars wanting to qualify accurately the theoretical content of the paper met many difficulties. Indeed, Curie's illustration is a mixture of metaphysics (e.g. references to "causes"), features of a deductive organization, problems, physical principles, and experimental rules. In the last 130 years, only two papers (Radicati, 1987; Castellani & Ismael, 2016) offered sketchy and incomplete summaries of the contents of Curie's paper.

As a premise, I indicate my method of investigation. Previous papers (Drago, 2023; 2024) showed that 1) the word "symmetry" is a double negation without a corresponding affirmative word (i.e. "Two negations do not affirm", DNP); hence, it belongs to intuitionist logic where the double negation law fails. 2) Recently a new theoretical organization which is an alternative to the deductive-axiomatic one (AO) of the Newtonian mechanics has been recognized (Drago, 2007); being based on a basic problem it is called a problem-based organization (PO); 3) The ideal model of a PO develops through four logical steps: *i*) to state a problem; *ii*) to argue through DNPs *iii*) composing *ad absurdum* arguments (AAAs); *iv*) whose general conclusion is translated into an affirmative proposition to be tested with reality in order to prove or not the entire theory. 4) In Curie's paper a metaphysical part represents an unsuccessful effort of philosophical generalization to a general theory. 5) The theoretical organization of his theory is not a deductive-axiomatic theory (AO); rather, it substantially is a PO.

In light of these novelties, I examine all the theoretical propositions of Curie's paper. I elucidate their contents and examine their logical, mathematical, and physical differences to establish their relations. Only two propositions are recognized as the correct versions of the principle: a mathematical inequality and a group theoretical formula; both belong to intuitionist logic, like the word "symmetry". The interpretation of the remaining theoretical propositions leaves some open questions.

Because the interpretation of Curie's paper was very difficult, first of all, scholars tried to extract from its theoretical propositions the most meaningful one(s), called "Curie's principle(s)" (CP). Its many versions are discussed in the following by examining their logical, mathematical, and physical differences. In order to avoid the difficulties someone bounded the theoretical framework of this principle to be a deterministic and temporal one. The remaining scholars recognized as Curie's principle one or more propositions without clarifying the set of the remaining ones. Only a few of them agreed with the intuitionist logic to which the word "symmetry" belongs. No discussion of the remaining theoretical propositions was offered. However, some authors approached a reconstruction of Curie's theory according to a problem-based theoretical organization.

In conclusion, there exists a theoretical proposition that can be called "Curie principle" and that it has to be applied as a heuristic principle within a problem-based theoretical organization.

2. Curie's theoretical propositions

Curie's paper includes 18 theoretical propositions that may be qualified as "principles" or elements of these principles. They all refer to not local, particular situations, but to general theoretical contexts. Almost all are located at the beginning of the paper (pp. 394, 400-401) and at the end (p. 414). In the following, I quote all of them. Among them, Curie emphasized 8 propositions in Italic in order to underline their basic role in his paper.

In order to understand them one has to take into account that the "elements of symmetries" which Curie refers to are e.g. a point, an axe, or a plane of symmetry (Curie, 1894, p. 394-396). Moreover, an asymmetry of a physical being is the lack of an element of symmetry in it. The lack of a symmetry producing symmetry has to be intended as Elena Castellani explains:

"for the occurrence of a phenomenon in a medium, the original symmetry group of the medium must be lowered (broken, in today's terminology) to the symmetry group of the phenomenon (or to a subgroup

of the phenomenon's symmetry group) by the action of some cause (the electric field and the torque in the above example). In this sense symmetry breaking is what *creates the phenomenon*" (Castellani, 2003, p. 324).

To name each Curie's proposition, I preface it with a letter of the Greek alphabet. Some explicative words are inserted between square brackets [ ]. The negative words composing a DNP are underlined to facilitate recognition by the reader; for the same reason, the modal words are point underlined.

> α *Enfin lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les [éléments de symétrie des] effets produits"* (p. 394).
> β *La symétrie caractéristique d'un phénomène est la symétrie maxime compatible avec l'existence du phénomène.*
> γ *Un phénomène* [symétrique] *peut exister dans un milieu qui possède sa* [même] *symétrie caractéristique ou celle d'un des intergroupes* [= sous-groupes] *de sa symétrie caractéristique*.
> δ Autrement dit, certains éléments de symétrie [du milieu] peuvent coexister avec certains [éléments de symétrie des] phénomènes, mais ils ne sont pas nécessaires. ϵ Ce qui est nécessaire, c'est que certains éléments de symétries [du phénomène] n'existent pas [dans le milieu].
> ζ *C'est la dissymétrie* [dans le milieu] *qui crée le* [la symétrie dans le] *phénomène.* (p. 400)
> η On peut encore voir que quand plusieurs phénomènes de natures différentes se superposent dans un même système, les [leurs éléments de] dissymétries s'ajoutent. θ Il ne reste plus alors comme éléments de symétrie dans le système que ceux qui sont communs à chaque phénomène pris séparément.
> ι *Lorsque certaines causes produisent certains effets* [symétriques], *les éléments de symétrie des causes doivent se retrouver dans les* [éléments de symétrie des] *effets produits.*
> κ *Lorsque certains effets révèlent une certaine dissymétrie, cette dissymétrie doit se retrouver dans les causes qui lui ont donné naissance*.
> λ La réciproque de ces deux propositions n'est pas vraie, au moins pratiquement, c'est-à-dire que les effets produits peuvent être plus symétriques [avoir plus éléments de symétrie] que les [éléments des] causes. Certaines causes de dissymétrie [du milieu] peuvent ne pas avoir d'action sur certains phénomènes ou du moins avoir une action trop faible pour être appréciée, ce qui revient pratiquement au même que si l'action [du milieu] n'existait pas (p. 401).
> μ Lorsque deux phénomènes de nature différente se superposent dans un même milieu, les dissymétries s'ajoutent (p. 409)
> ...Au point de vue des applications, nous voyons que les conclusions que nous pouvons tirer des considérations relatives à la symétrie sont de deux sortes: Les premières sont des conclusions fermes mais négatives, elles répondent à la proposition incontestablement vraie: ν *Il n'est pas d'effet* [symétrique] *sans* [dissymétrie des] *causes*. ξ Les effets, ce sont les [éléments de symétrie du] phénomènes qui nécessitent toujours, pour se produire, une certaine dissymétrie [des éléments du milieu]. o Si cette dissymétrie n'existe pas [dans les éléments de symétrie du milieu], le [la corréspondante symétrie du] phénomène est impossible.
> π ...une deuxième sorte de conclusions, celles-ci de nature positive, mais qui n'offrent pas la même certitude dans les résultats que celles de nature négative. Elles répondent à la proposition: *Il n'est pas de* [dissymétrie dans la] *cause sans effets* [symétriques]. ρ Les effets [symétriques], ce sont les phénomènes [possédant symétries] qui peuvent naître dans un milieu possédant une certaine dissymétrie; on a là des indications précieuses pour la découverte de nouveaux phénomènes; mais les prévisions ne sont pas des prévisions précises comme celles de la Thermodynamique. On n'a aucune idée de l'ordre de grandeur des phénomènes prévus : on n'a même qu'une idée imparfaite de leur nature exacte. σ Cette dernière remarque montre qu'il faut se garder de tirer une conclusion absolue [= pas relative] d'une expérience négative (p. 414).

3. Curie's theoretical propositions: explanations and their formalizations

To put a remedy to the obscure Curie's language, in the following I explain each theoretical proposition and translate it into a corresponding mathematical or logical formula, which is added in curly brackets[2].

α is an ambiguous proposition because the metaphysical language ("causes"[3], "produisent") makes not easy the comprehension of its meaning. Notice that the possible case $SE(c) = SE(e)$ is not explicitly considered.

$\{M : \Box(SE(c) < SE(e))\}$.

β is obscure because the modal word ("compatible") does not give an accurate idea of what is declared and because the "existence of a phenomenon" is defined by the next proposition.

$\{M : SE(p) = maxSE(p) : \Diamond\exists SE(p)(= (SE(p) \geq SE(m))\}$

γ clarifies the content of β in terms of groups and subgroups. A good order of the propositions was γ before β.

$\{M : (G(p) \supseteq G(m)) \rightarrow \Diamond\exists S(p)\}$

δ is obscure because it is not clear to what refer the mentioned elements of symmetries. I fix their meanings by means of some words put within square brackets. One may better say that all the elements of symmetry of the medium can be less than that of the phenomenon.

$\{M : \Diamond(SE(p) \geq SE(m))\}$

ϵ is clear once one has referred to the words "symétries" to two physical subjects of the proposition. Oddly enough, instead of explaining the previous one, this proposition enhances it as a necessity: it is necessary that the elements of symmetry of the phenomenon are more than those of the medium. But this is not true, because it excludes the possible case of equality of the two sets of elements of symmetries as in β, δ and γ; otherwise one has to replace "possible" for "nécessaire".

$\{M : \Box(SE(m) < SE(p))\}$

Curie declares both propositions δ and ϵ equivalent ("Autrement dit,..") to the previous proposition γ. Truly, their formal translations make manifest that they are all modal propositions but they are not equivalent; because γ states a relation that is compatible with δ, but not with ϵ which denies the case of equality, allowed by both γ and δ. However, these three propositions become mutually equivalent if one changes ϵ as suggested before. This point manifests a Curie's fault (in the following sect. we will see a different interpretation).

ζ wants to repeat the content of the previous three propositions, in particular ϵ, through the asymmetries (called by him "dissymétries") which are considered as the necessary causes of the phenomenon's symmetry. However, the verb "crée" is emphatic because without any operative support; it qualifies in metaphysical terms of "cause-effect" the relation between the asymmetry of the medium and the existence of the phenomenon. (Notice that in this context the word "dissymétrie" is not a negation; hence this proposition is not a DNP).

$\{M : aSE^{\circ}(m) \rightarrow \Box\exists SE(p)\}$

η adds a rule on asymmetries. Its first part (once one replaces "les" with "leurs") is a clear (affirmative) proposition.

$\{\text{Aff} : aSE(s) = \cup_i aSE(p_i)\}$

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[2] In the following, s means the system, c the cause, e the effect, m the medium, p the physical phenomenon, $G(m)$ and $G(p)$ the symmetry group of respectively the medium and the phenomenon. $SE(c)$, $SE(e)$, $SE(m)$, $SE(p)$ and $SE(s)$ are the sets of symmetry's elements of respectively cause, effect, medium, phenomenon, and system, The symbol aSE stands for the elements of an asymmetry. The symbol $^{\circ}$ means a single element of (a)symmetry. The symbol c(e) is the function cause-effect, and e(c) is the inverse function. The symbol $\Box$ stands for necessity and $\Diamond$ for possibility. Moreover, Aff stands for an affirmative proposition and M for a modal proposition. The symbol "$\supseteq$" denotes the equality or the inclusion.

[3] Roche (1987, p. 22) remarks that not all symmetries are causal; e.g. "Mariotte argument, which "from a geometrical symmetry of a vibrating string [concludes] its emotional symmetry" deals with correlated properties only."

θ comes back to symmetries. (The following words "communs à chaque phénomène pris séparément" actually mean "communs à tous les phénomènes". The proposition is not a DNP because it is equivalent to the corresponding affirmative proposition).
$\{\text{Aff} : aSE(s) = \cap_i aSE(p_i)\}$
ι *verbatim* reiterates α.
$\{M : \Box(SE(c) < SE(e))\}$
κ again does not explicitly consider the case $aSE(c) = aSE(e)$
$\{M : \Box(aSE^{\circ}(e) < aSE^{\circ}(c)\}$
λ is declared the "réciproque" (counter-nominal) of propositions ι and κ (they are implicitly considered as mutually equivalent). But Curies declares that λ is not true, at least owing to the "pratique" limitation presented by the last proposition. But then, the second part of the first proposition ("c'est-à-dire") does not refer to the previous one, merely reiterates $\iota(= \alpha)$. It is the next proposition ("Certaines causes") that explains Curie's limitation: some asymmetries of the medium (called "causes de dissymétrie" of the phenomenon) are not effective in generating detectable symmetries within the phenomenon. Actually, this practical limitation does not deny ι and α; it merely adds one more reason for the possible inequality between $aSE(e)$ and $aSE(c)$ or their respective asymmetries established by δ and ϵ.
$\{M : \Diamond\exists aSE^{\circ}(c) \wedge \vdash SE^{\circ}(e)\}$
μ reiterates in the case of two asymmetries previous rule η on the addition of the asymmetries of effects.
$\{\text{Aff} : aSE(s) = aSE(p_1) + aSE(p_2)\}$
ν tries a synthesis of the previous theoretical proposition by referring to the metaphysics of causes and effects. (Curie calls "negative" this proposition which actually is a DNP. The following formula reiterates that of the principle of sufficient reason: $\vdash\exists x \vdash f(x)$)
$\{DNP : \vdash\exists SE(e) \vdash c(e) (= aSE(c)\}$
ξ is essentially the same proposition as ζ and ϵ (whose relation medium-phenomena is replaced by that causes-effects)
$\{M : \Box(\exists SE^{\circ}(p) \rightarrow \exists aSE^{\circ}(m))\}$
o is the double negation of ζ and ξ.
$\{M\&DNP : \vdash\exists aS^{\circ}(m) \rightarrow \vdash\Diamond\exists SE(p)\}$
π is the inverse implication of proposition ν.
$\{DNP : \vdash\exists SE(e) \vdash c(e) (= aSE(c)\}$
ρ is a crucial result of Curie's paper: π does not correspond in logical terms to an inversion of ν owing to the practical limitation according to λ of the proposition $aSE(m) \rightarrow SE(p)$
σ is advice on the previous heuristic previsions obtained by the above rule. σ does not give an absolute value to the previous laws
$\{M : \Diamond \vdash (aSE(m) \rightarrow SE(p))\}$

Medium

S	A	A
$\alpha \downarrow$	$\delta \downarrow \epsilon, \lambda \uparrow$	$\kappa \uparrow$
S		A

Phenomenon

Tab. 1: A symmetry. S = symmetry. $\rightarrow$ = implication.

4. Analysis of Curie's theoretical propositions

Curie's paper makes use of metaphysical words, modal words, and DNPs. Their uncommon mixture made difficult the recognition of the scientific contents of the text for scholars preconceived by classical logic. Let us accurately examine its 18 propositions.

The text includes 8 metaphysical propositions: the proposition ζ contains the word "créer" and the propositions $\alpha, \iota, \kappa, \lambda, \nu, \xi, \pi$ and ρ contain the words "cause" and/or "effect". A previous paper (Drago, 2024) showed that Curie's use of a couple of words causes-effects tries to generalize into metaphysics what is a physical relation between medium-phenomenon; but unsuccessfully. Therefore, one can avoid the metaphysics of the text by replacing the former couple of words with the latter one.[4].

As a consequence of these two remarks, one can conclude that owing to the radical differences in metaphysics (and in logic) it is unlikely that the various theoretical propositions of Curie's paper can be summarized by one of his propositions. The search for the wanted unique CP has to accurately interpret the text.

Now let us search for the main content of Curie's paper. First of all, we discard the less relevant propositions. The propositions ρ ("les previsions ne sont pas... précises") and σ ("ne pas tirer une conclusion d'une experience negative") are additions to the scientific text in order to warn the reader on how to manage previous theoretical results. They may be disregarded for our purpose. Each of them includes two negations but we can verify that the contents of these propositions are equal to the corresponding affirmative ones; hence, here Curie applies classical logic.

Propositions η, θ, and μ concern lateral results, i.e. the results of how either asymmetries (η and μ) or symmetries (θ) overlap. Proposition η (concerning asymmetries) implies its negative proposition θ (concerning symmetries), and moreover, as a particular case of overlapping the asymmetries of two phenomena, proposition μ. Notice that these propositions do not leave uncertainties; they are affirmative propositions as those which usually state scientific results, i.e. they belong to classical logic.

The number of the remaining propositions is 13; but some propositions are almost the same: $\alpha = \iota$ and $\epsilon \approx \zeta \approx \xi \approx o$. Hence, we have to investigate the contents of the remaining 9 propositions. Apart δ, ϵ, and λ, they are all highlighted by Curie in Italics, apparently for summarizing the entire theoretical part of his paper.

Let us consider the first 4 propositions of the list of section 2 (α, plus κ (on asymmetries). By including the case $SE(m) = SE(p)$, the propositions β, γ, and δ are the clearest ones. As a fact, these propositions work as methodological principles for developing a great part of his analysis. Among them, the clearest proposition is γ, also because it makes use of the modern language of groups:
$\{G_p \supseteq G_m \rightarrow \Diamond \exists S(p)\}$. Notice that it, as well as δ, includes the case of equality, hence the following propositions of the above list are no longer considered. Apparently, γ is the general principle while the other propositions illustrate some specific consequences. Notice that γ (which is equivalent to a DNP

[4] Rather, it is surprising that (except for 5 propositions: η, θ, μ, ν and π) 13 theoretical propositions are modal. Beyond an affirmation, a modal proposition offers the easiest way to mentally grasp the content of a proposition; it seems to communicate a subjective feeling of it. Just for this subjective aspect a modal word softens or blurs the meaning of its proposition intended by a classical logic-minded reader; therefore, modal words are inappropriate for establishing metaphysical laws concerning "causes"; and even less for establishing experimental laws, both about classical logic. Hence, so many modal words within Curie's scientific paper communicate at first sight a feeling of insecurity. However, it is a remarkable result of mathematical logic that modal logic is equivalent, *via* its S4 model, to intuitionist logic (Hughes & Cresswell, 1996, pp. 224); hence a modal word represents a DNP; but, being its meaning also of a subjective nature, a modality is less accurate than a DNP; for ex. "It is possible" is logically equivalent to "It is not true that it is not" but its meaning is more vague of the latter. Indeed, modal logic is not well-defined (Grason, 2023, preface and Sect. 1) whereas intuitionist logic has been completely formalized by Kolmogorov in 1932 (Drago, 2021). Let us note that sometimes to enhance the meaning of a DNP the common language adds to such proposition a modal word, as in the proposition "It is impossible a motion without an end"; where the appeal to a (im)possibility adds a reinforcement to its meaning. That occurs in the propositions γ and O.

because the group theoretical formula expressing it, $G_p \supseteq G_m$, is not a pure equality) includes also a modal word ("peut"). Hence, all previous propositions are modal. Here Curie applies modal logic.

Previous propositions constitute kinematics; the words "cause" and "effect" merely leave room for imaginary dynamical processes. Instead, from here on, Curie's theory wants to illustrate through 4 propositions the dynamics of the process generating symmetries. This dynamics may be conceived in a parallel way to the introduction into theoretical mechanics of dynamics expressed by the cause-effect relation $F = ma$ between force F and acceleration.

Let us now examine the proposition $\epsilon \approx \zeta \approx \xi \approx o \approx \{\Box SE(m) < SE(p)\}$. This is the more obscure point of the paper because no general mechanism of symmetry production is suggested by Curie, but only verbal propositions which are also metaphysical in nature, without examining if they are mutually compatible or not. Unfortunately, this subject occurs in (two locations of) the middle of the paper; so that the reader cannot understand the contents of the paper without solving this point.

In the previous section the comment to proposition ϵ suggested that this proposition is ambiguous: "possibility" may replace "necessity". However, this ambiguity is solved by the next proposition ζ of the list and then ξ and O; Curie wanted to write the troubling word "necessary". Therefore, it is clear that here Curie considers the process generating a new symmetry phenomenon from an already symmetric situation; his sentence states that this process is necessarily caused by an asymmetry. (Within the paper this subject is treated in p. 407, almost at the end). It is apparent that this proposition is not in agreement with both γ and the propositions of its group, which all do not exclude the equality case $SE(m) = SE(p)$. In particular, λ states a limitation of the process of generating asymmetries. The ideal situation of causes (asymmetries) corresponding one-to-one to effects' symmetries is denied, due to practical considerations about an insufficient detection of a symmetry phenomenon. Therefore, the proposition λ states that not always to an asymmetry of the medium correspond a symmetric phenomenon: $\diamond\vdash(aSE^{\circ}(m) \rightarrow SE^{\circ}(p)$. Hence, the novelty is rather explained by the introduction of a symmetry production or practical considerations (undetectable phenomena of symmetry). So the paper includes two subjects, the recognition of the symmetries of a phenomenon inside a medium and the "creation" of new phenomena of symmetry from previous others. However, the trespassing from kinematics to dynamics remains unresolved also because Curie tried to formulate it in metaphysical terms. Notice that the above propositions are modal; also here Curie applies modal logic.

Through the remaining 2 propositions, ν and π Curie wants to summarize the results of the entire paper; they concern a relationship of ontological metaphysics, cause-effect. But, by making essential use of DNPs he formulates them within henological metaphysics (the search of unity).

Let us now examine Curie's "conclusions"; they may be translated into the following logical formulas:
$\{\vdash\exists e\vdash c(e); \vdash\exists c\vdash c(e)\}$
where c stands for "cause" and e for "effect"[5]. In classical logic both implications, the direct and inverse ones have to hold true. Instead, we see that also in the simpler logical case (i.e. in propositional calculus: $e \rightarrow c$ and $c \rightarrow e$), proposition λ states that sometimes $c \rightarrow e$ fails, i.e. asymmetry of the medium does not always gives a symmetry of phenomenon:
$\{\diamond\vdash(aSE^{\circ}(m)) \rightarrow \exists SE^{\circ}(p))\}$. Only the former implication $\vdash\exists e\vdash c(e)$ is valid; however, it represents the lesser interesting one (effect-cause) from the (ontological) metaphysical viewpoint. Hence, the general relation between c and e cannot be the specific logical relation through which traditional metaphysics represents the connection between cause and effect, i.e. a logical equivalence. Hence the kind of logic of Curie's paper is necessarily the intuitionist one.

[5] The latter "Conclusion" is called by Curie the "reciprocal" of the former one, "converse" by (Castellani & Ismael, 2016, p. 1003) and also by others.

But this relation is intuitionist also because the set of symmetries of the phenomenon may be greater than the set of symmetries of the medium. That can be translated into either the group theoretical formula γ or a mathematical inequality: $S(p) \geq S(m)$. Ultimately, the entire difficulty of expressing Curie's first theoretical proposition in mathematical language boils turns out to writing *an inequality instead of* the equalities = obtained by the translations of usual physical principles into mathematical formulas.

We know that in thermodynamics there exists an inequality translating the principle of the increase of entropy: $\Delta S \geq 0$. Unfortunately, Curie ignored this formula because it was enlightened by Max Planck in 1897, three years after Curie's paper. Do not consider irrelevant Curie's failure. The occurrence of entropy's inequality within theoretical physics led philosopher Émile Meyerson (1908) to see in $\Delta S \geq 0$ the defeat of scientific reason, whose arguing and results have been always expressed through equalities (and hence classical logic)[6].

Here is the core of Curie's theory; it belongs almost entirely to intuitionist logic. All that confirms that being a double negation, the word "symmetry" leads to laws that pertain to intuitionist logic and are expressed by mathematical inequalities.

5. The interpretations of Curie's paper through a "Curie's principle"

Since the interpretation of Curie's paper was difficult, the scholars tried to grasp the main content of the paper by extracting from the above theoretical propositions the most meaningful one(s). Hence, they tried to pick up the most representative proposition(s) and called it (or them) "Curie principle(s)" (from which deriving all consequences according to an AO). But also this task met great difficulties. As a historical result, the scholars offered many versions of it. In the literature, I have found 21 versions (with some repetitions) of CP. Hence, the scholars do not agree on what CP is[7].

Many authors present more than one principle: van Fraassen (1989) 2 principles, Nakamura & Nakahama (2000) 3, Brading *et al.* (2023) 2, Castellani & Ismael (2016) (whose title is exactly "Which Curie principle?") 5. The remaining 17 scholars 1 proposition. Hence, scholars disagree even on the number of the propositions composing Curie's principles.

Furthermore, they differ in their philosophical assumptions (either a deterministic framework or a relation medium-phenomenon, or a relation problems/solutions), in their mathematics (either inequalities or a theoretical group's formula), and in their kind of logic (either classical logic or DNPs of intuitionist logic or modal logic). These great differences manifest the scholars' embarrassment in verbally circumscribing one of Curie's principles, although the original text is a century old.

Moreover, from no version of CP it is possible to derive within an AO all Curie's theoretical propositions as its consequences. To avoid the metaphysics of causes and reduce the theory to the usual framework of theoretical physics like Newtonian mechanics, Chalmers in 1970 and later Roberts and Earman, 2002 immersed Curie's propositions in an a priori mathematical and deterministic framework equipped by a parameter time. Then CP becomes: "A system cannot evolve from a symmetric to an asymmetric state." However, the authors do not specify what of Curie's original thoughts they leave outside and whether they make violence to the text.

All that confirms the conclusion of (Drago, 2024): CP, as a single principle-axiom, cannot start a re-construction of Curie's theory as an AO because Curie's theory represents a mixture of an AO and a PO.

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[6] Also this philosophical difficulty in the next decades led many theoretical physicists to resist to the introduction into theoretical physics of symmetries which actually implied inequalities (Drago, 2023).

[7] Most authors preserve Curie's modalities or add new modalities.

Without examining the results of all scholars, let us now analyze an example of a definition of CP given by the authoritative *Stanford Encyclopedia of Philosophy*, already quoted in footnote no. 1. This definition is merely a collection of Curie's propositions; its propositions (a) is essentially γ and ϵ; the proposition (b) is essentially ζ, α and λ. The authors (and also Castellani & Ismael, 2016, p. 1003) state that "propositions a and b" are not independent of one another". But since only $\epsilon \approx \zeta$, the equivalence of (a) and (b) concerns only two of the four propositions. At last, these authors consider as CP only the proposition α: "[It] Is what has become known as Curie's principle (CP)". But this definition misses some facts stated by Curie: the effects can have the same symmetries of the causes, the translation of γ (which is a DNP) into group theoretical language and the practical limitation illustrated by λ. However, to be faithful to the text's contents, one must add at least the propositions $\zeta, \iota, \kappa, \lambda, \nu, o, \pi$, and ρ. As a fact, not all scholars agree on this interpretation.

It is important to note that according to a suggestion by (Birkhoff, 1950, p. 30 and 45), van Fraassen and Castellani change the original words "causes" and "effects" into respectively the words "problems" and "solutions". This rewording removes Curie's metaphysics. Then the new text implicitly raises a *problem* (Which solutions-symmetries?) and its theory has to be intended as a search for discovering a new method for *solving* it. Moreover, Castellani & Ismael (2016, p. 1006) suggested that CP is not an axiom but a methodological or heuristic principle, exactly as it has to be within a PO. Therefore, this interpretation of Curie's paper implicitly conforms to the theoretical organization PO; it puts the basis for reconstructing Curie's paper as a PO theory of the first group of the 18 Curie's propositions[8].

In conclusion, once the PO is chosen, CP receives a plain expression in DNPs: either the group theoretical formula of the proposition γ or the inequality repeating Curie's main proposition α ("Lorsque") without its metaphysics of causes and its modal word "doivent": "The number of symmetries of the problem is lesser than or equal the number of symmetries of the solutions".

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[8] Whereas the question of which is the dynamics of symmetry's generation is avoided. How to obtain a complete PO theory from these first elements of this theoretical organization? This question will be met in a successive paper.

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